T&T3 3.2

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PROJECT MATHS

Text & Tests 3

Leaving Certificate

Coordinate Geometry – The Line

chapter 3

Key words

Cartesian plane origin axis quadrant vertex horizontal
vertical slope parallel perpendicular positive negative
linear equation area translation intersection collinear

Section 3.2 Distance between two points

Formula Log Tables Pg 18

$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

mean Distance

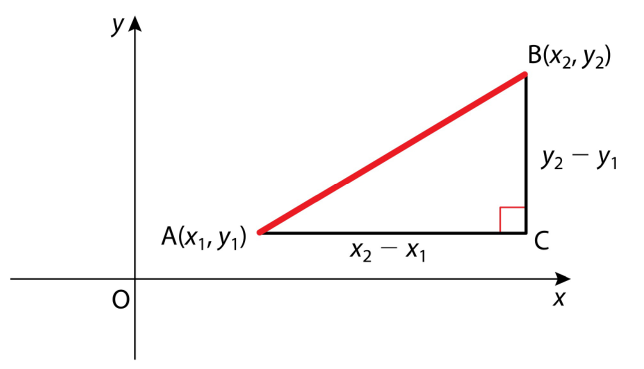
Method : 1) Label the two points (x_1,y_1) (x_2,y_2)
2) Sub the labelled values in the formula

Notes

Section 3.2 Distance between two points

The given diagram shows the points A(x_1, y_1) and B(x_2, y_2).

$|BC| = y_2 - y_1$ and $|AC| = x_2 - x_1$



Using the Theorem of Pythagoras:

$$|AB|^2 = |AC|^2 + |BC|^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\therefore \quad |\mathbf{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

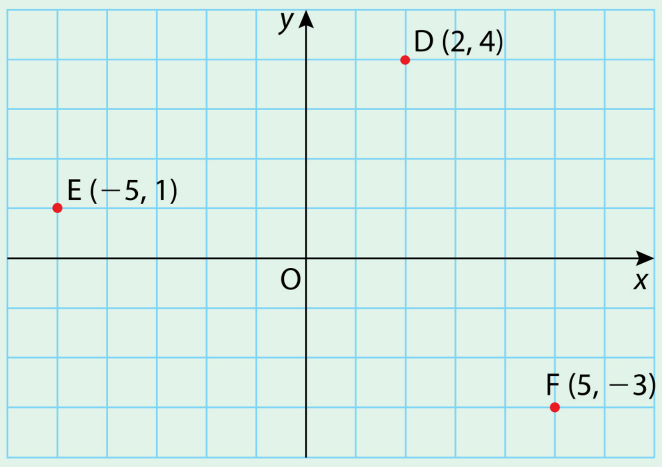
The distance between A(x_1, y_1) and B(x_2, y_2) is $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1

Show that D(2, 4) is equidistant from E(−5, 1) and F(5, −3).

Equidistant means the same distance.

The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is
$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



D(2, 4) → (x_1, y_1) E(−5, 1) → (x_2, y_2)

$$|DE| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-5-2)^2 + (1-4)^2}$$
$$= \sqrt{(-7)^2 + (-3)^2}$$
$$= \sqrt{49 + 9} = \sqrt{58}$$

D(2, 4) → (x_1, y_1) F(5, −3) → (x_2, y_2)

$$|DF| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-2)^2 + (-3-4)^2}$$
$$= \sqrt{(3)^2 + (-7)^2}$$
$$= \sqrt{9 + 49} = \sqrt{58}$$

Since $|DE| = |DF| = \sqrt{58}$, D is equidistant from E and F.

Example 2

If the distance between the points (2, 3) and (5, k) is $\sqrt{10}$, find two possible values of k.

The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is
$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(x_1, y_1) → (2, 3) (x_2, y_2) → (5, k)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-2)^2 + (k-3)^2}$$
$$= \sqrt{9 + k^2 - 6k + 9}$$
$$= \sqrt{k^2 - 6k + 18}$$

$$\text{Distance} = \sqrt{10} \Rightarrow \sqrt{k^2 - 6k + 18} = \sqrt{10}$$
$$\Rightarrow k^2 - 6k + 18 = 10$$
$$\Rightarrow k^2 - 6k + 8 = 0$$
$$\Rightarrow (k-2)(k-4) = 0$$
$$\Rightarrow k = 2 \quad \text{or} \quad k = 4$$

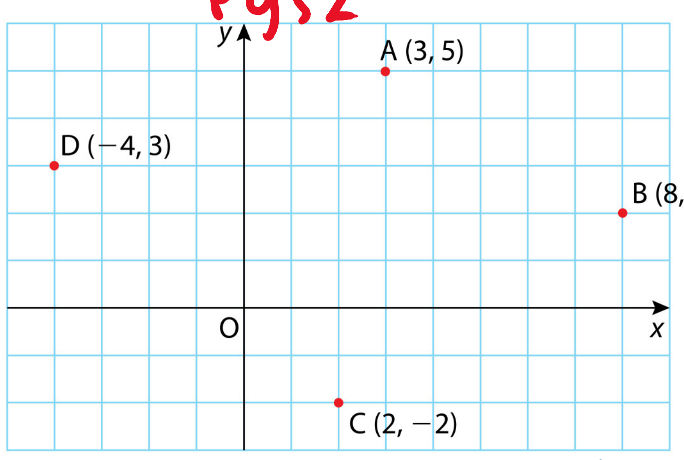
Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Pg52

1. The points A, B, C and D are shown.
   Find (i) |AB|
   (ii) |AC|
   (iii) |AD|.
   Is |DC| = |BC|?

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



1) $A(\overset{x_1}{3},\overset{y_1}{5})\ B(\overset{x_2}{8},\overset{y_2}{2})$ |AB|

$|AB| = \sqrt{(8-3)^2 + (2-5)^2}$

$\sqrt{(5)^2 + (-3)^2}$

$\sqrt{25 + 9}$

$\sqrt{34}$ surd [SD]

Decimal 5.831

ii) $A(\overset{x_1}{3},\overset{y_1}{5})\ C(\overset{x_2}{2},\overset{y_2}{-2})$

$|AC| = \sqrt{(2-3)^2 + (-2-5)^2}$

$\sqrt{(-1)^2 + (-7)^2}$

$\sqrt{+1 + 49}$

$\sqrt{50}$ surd form

$= 7.071$

iii $A(\overset{x_1}{3},\overset{y_1}{5})\ D(\overset{x_2}{-4},\overset{y_2}{3})$

$|AD| = \sqrt{(-4-3)^2 + (3-5)^2}$

$\sqrt{(-7)^2 + (-2)^2}$

$\sqrt{49 + 4}$

$\sqrt{53}$

$= 7.280$

$|DC| = D(\overset{x_1}{-4},\overset{y_1}{3})\ C(\overset{x_2}{2},\overset{y_2}{-2})$

$\sqrt{(2+4)^2 + (-2-3)^2}$

$\sqrt{6^2 + (-5)^2}$

$\sqrt{36 + 25}$

$= \sqrt{61}$

$|BC| = B(\overset{x_1}{8},\overset{y_1}{2})\ C(\overset{x_2}{2},\overset{y_2}{-2})$

$\sqrt{(2-8)^2 + (-2-2)^2}$

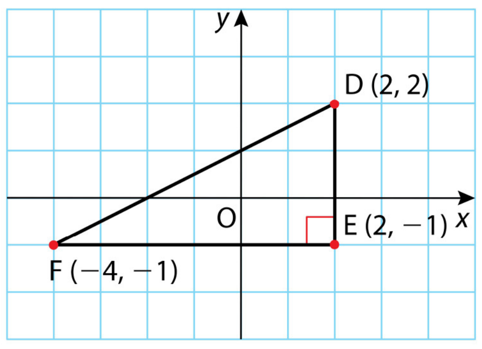
$\sqrt{(-6)^2 + (-4)^2}$

$\sqrt{36 + 16}$

Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. The given diagram shows the points D, E and F.
(i) Write down the lengths of [FE] and [ED].
(ii) Find $|DF|$.
Use the Theorem of Pythagoras to show that the triangle DEF is right-angled.



$\sqrt{52}$

$|BC| \neq |DC|$

$\sqrt{52} \neq \sqrt{61}$

Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

H/w

3. Find the distance between each of the following pairs of points:
(i) (2, 1) and (3, 4)
(ii) (1, 5) and (2, 3)
(iii) (−1, 4) and (2, 6)
(iv) (3, −2) and (−5, 3)
(v) (−6, −1) and (1, −3)
(vi) (4, −2) and (0, −5)

Pg 53 Q3 H/W.

Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

4. Find $|AB|$ in each of the following:
(i) A = (2, −4), B = (3, 1)
(ii) A = (0, 3), B = (−2, 5)
(iii) A = (0, −2), B = (3, −1)
(iv) A = (5, −2), B = (3, −4)

Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

5. A(1, 1), B(3, 6) and C(5, 1) are the vertices of a triangle. Show that $|AB| = |BC|$.

Exercise 3.2

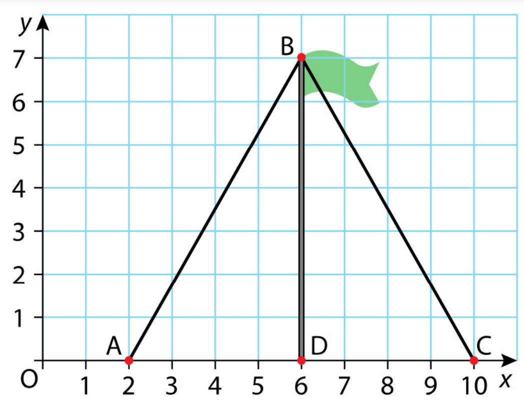
$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

6. X(1, 6), Y(−3, −1) and Z(2, −2) are the vertices of a triangle.
Find the lengths of the 3 sides and then state which two sides are equal in length.
Hence state what type of triangle is XYZ.

Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

7. A wire ABC is used to support a flag pole [BD], as shown on the right.
Write down the coordinates of A, B, C and D.
Calculate the length of wire needed to support the pole.



$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

8. The centre of a circle is $(-3, 1)$ and $(4, 3)$ is a point on the circle.
Find the length of the radius of the circle.

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

9. The points $A(2, 1)$, $B(6, 1)$, $C(5, -2)$ and $D(1, -2)$ are the vertices of a parallelogram.
Plot the parallelogram on a coordinated plane.
Find (i) $|AC|$ (ii) $|BD|$.
Are the diagonals equal in length?

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

10. The distance between the points $(5, 2)$ and $(4, k)$ is $\sqrt{2}$.
Find two possible values for k.

Exercise 3.2

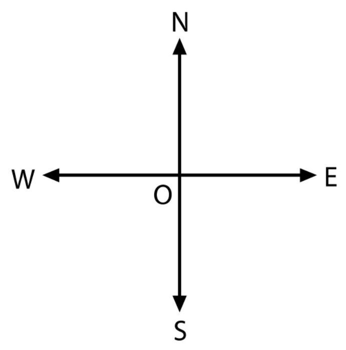
$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

11. X(3, k) and Y(-1, 2) are two points.
If $|XY| = 5$, find two possible values for k.

Exercise 3.2

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

12. Jordan lives (3 km West, 4 km South) of the centre of the town marked O in the given diagram.
Michelle lives (2 km West, 3 km North) of Jordan's house.
How far does Michelle live from the centre of town?



Exercise 3.2 Answers

1. (i) $\sqrt{34}$ (ii) $\sqrt{50}$ (iii) $\sqrt{53}$; No
2. (i) $|FE| = 6$, $|ED| = 3$ (ii) $\sqrt{45}$
3. (i) $\sqrt{10}$ (ii) $\sqrt{5}$ (iii) $\sqrt{13}$
(iv) $\sqrt{89}$ (v) $\sqrt{53}$ (vi) 5
4. (i) $\sqrt{26}$ (ii) $\sqrt{8}$ (iii) $\sqrt{10}$ (iv) $\sqrt{8}$
6. $|XY| = \sqrt{65}$; $|XZ| = \sqrt{65}$; $|YZ| = \sqrt{26}$;
$|XY| = |XZ| \Rightarrow \triangle XYZ$ is isosceles
7. A(2, 0), B(6, 7), C(10,0), D(6,0); $2\sqrt{65}$ units
8. $\sqrt{53}$
9. (i) $\sqrt{18}$ (ii) $\sqrt{34}$; No
10. $k = 1$ or $k = 3$
11. $k = 5$ or $k = -1$
12. $\sqrt{26}$ km

Answers