



PROJECT MATHS

Text & Tests

Leaving **3** Certificate

Algebra 1

chapter

1

Section 2.3 Solving quadratic equations involving fractions

Notes

To solve an equation which contains fractions, we multiply each term of the equation by the smallest number into which each of the denominators divide (i.e. the LCM of the denominators). This often results in a quadratic equation as the following examples illustrate.

30

Example 1

Solve the equation $\frac{x-3}{3} + \frac{12}{x} = 4$.

The LCM of 3 and x is $3x$.

We now multiply each term by $3x$.

$$\frac{3x(x-3)}{3} + \frac{3x(12)}{x} = 4(3x)$$

$$\therefore x(x-3) + 3(12) = 4(3x)$$

$$\therefore x^2 - 3x + 36 = 12x$$

Take $12x$ from each side: $x^2 - 3x + 36 - 12x = 12x - 12x$

$$x^2 - 15x + 36 = 0$$

Factorise: $(x-3)(x-12) = 0$

$$\therefore x-3 = 0 \text{ or } x-12 = 0$$

$$\therefore x = 3 \text{ or } x = 12$$

30

Example 2

Solve the equation $\frac{2}{x-1} - \frac{1}{x+2} = \frac{1}{2}$

The LCM of the denominators is $2(x-1)(x+2)$.

We now multiply each term by $2(x-1)(x+2)$.

$$\frac{2(2)(x-1)(x+2)}{(x-1)} - \frac{2(x-1)(x+2)}{(x+2)} = \frac{1 \cdot 2(x-1)(x+2)}{2}$$

$$4(x+2) - 2(x-1) = (x-1)(x+2)$$

$$4x + 8 - 2x + 2 = x^2 + 2x - x - 2$$

$$2x + 10 = x^2 + x - 2$$

$$2x^2 + x + 12 = 9$$

$$\therefore x^2 - x - 12 = 0 \dots \text{multiply each term by } -1$$

$$\therefore (x-4)(x+3) = 0 \dots \text{factorising}$$

$$\therefore x-4 = 0 \text{ or } x+3 = 0$$

$$\therefore x = 4 \text{ or } x = -3$$

30

Exercise 2.3

Solve each of the following equations:

1. $\frac{x}{x-5} + \frac{4}{x} = 0$

$LCD = x$

$$x(x) - 5(x) + 4(1) = 0$$

$$x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0 \begin{matrix} -4x \\ -1x \\ -5x \end{matrix}$$

$$\begin{matrix} x-1=0 & x-4=0 \\ \hline x=1 & x=4 \end{matrix}$$

31

Exercise 2.3

Solve each of the following equations:

2. $x - 7 + \frac{12}{x} = 0$

Exercise 2.3

Solve each of the following equations:

3. $\frac{x+7}{2} + \frac{2}{x} = 4$

LCD = (3)(x)

$x(x+7) + 3(2) = 3x(4)$

$x^2 + 7x + 6 = 12x$

$ax^2 + bx + c = 0$

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x - 3 = 0$

$x - 2 = 0$

$\Rightarrow x = 3$

$\Rightarrow x = 2$

Exercise 2.3

Solve each of the following equations:

4. $\frac{15}{x} + 2 = x$

Exercise 2.3

Solve each of the following equations:

5. $\frac{2x}{2x-1} + x = 3$ LCD = $2x-1$

$2x(1) + x(2x-1) = 3(2x-1)$

$2x + 2x^2 - 1x = 6x - 3$
 $2x^2 + 1x + 3 = 6x$
 $2x^2 - 5x + 3 = 0$

$(2x - 3)(x - 1) = 0$

$2x - 3 = 0$ $x - 1 = 0$
 $2x = 3$ $x = 1$
 $x = \frac{3}{2}$

$ax^2 + bx + c = 0$
 C/W
 Pg 31
 Q2, 4, 6, 7.
 Finish for
 H/W.

Exercise 2.3

Solve each of the following equations:

6. $\frac{x+1}{3} - \frac{1}{x} = 1$

31

Exercise 2.3

Solve each of the following equations:

7. $\frac{1}{x+1} + \frac{x}{5} = 1$

31

Exercise 2.3

Solve each of the following equations:

8. $\frac{4}{x} + \frac{1}{x-1} = 3$

31

Exercise 2.3

Solve each of the following equations:

9. $\frac{6}{x} - \frac{5}{x+1} = 2$

31

Exercise 2.3

Solve each of the following equations:

10. $\frac{5}{x-2} - \frac{3}{x+2} = 2$

31

Exercise 2.3

Solve each of the following equations:

11. $\frac{2}{x-2} + 3 = \frac{1}{x}$

31

Exercise 2.3

Solve each of the following equations:

12. $\frac{5}{2x-1} + 1 = \frac{6}{x}$

31

Exercise 2.3

Solve each of the following equations:

13. $\frac{9}{x+8} + \frac{1}{x} = 1$

31

Exercise 2.3

Solve each of the following equations:

14. $\frac{2}{x-1} + \frac{3}{x-3} = 2$

31

Exercise 2.3

Solve each of the following equations:

15. $\frac{5}{x-4} - \frac{2}{x-2} = 2$

31

Exercise 2.3

Solve each of the following equations:

16. $\frac{5}{x+2} + \frac{4}{x-1} + 6 = 0$

31

Exercise 2.3

Solve each of the following equations:

17. $\frac{5}{2x+1} + \frac{6}{x+1} = 3$

31

Exercise 2.3

Solve each of the following equations:

18. $\frac{3x+3}{x-1} = \frac{1}{x} + 1$

31

Exercise 2.3

19. Each of these equations has only one root. Find it in each case.

(i) $\frac{2x}{x+3} + \frac{1}{x} = 2$

31

Exercise 2.3

19. Each of these equations has only one root. Find it in each case.

(ii) $\frac{2x}{2x+1} + \frac{1}{x-1} = 1$

31

Exercise 2.3

19. Each of these equations has only one root. Find it in each case.

(iii) $\frac{1}{x+1} + \frac{x}{x-1} = 1$

31

Exercise 2.3 Answers

1. $x = 1$ or $x = 4$
2. $x = 3$ or $x = 4$
3. $x = 2$ or $x = 3$
4. $x = -3$ or $x = 5$
5. $x = \frac{3}{2}$ or $x = 1$
6. $x = -1$ or $x = 3$
7. $x = 0$ or $x = 4$
8. $x = \frac{2}{3}$ or $x = 2$
9. $x = \frac{3}{2}$ or $x = -2$
10. $x = -3$ or $x = 4$
11. $x = \frac{2}{3}$ or $x = 1$
12. $x = 1$ or $x = 3$
13. $x = -2$ or $x = 4$
14. $x = 5$ or $x = \frac{3}{2}$
15. $x = \frac{3}{2}$ or $x = 6$
16. $x = \frac{1}{2}$ or $x = -3$
17. $x = 2$ or $x = -\frac{2}{3}$
18. $x = -1$ or $x = -\frac{1}{2}$
19. (i) $x = \frac{3}{5}$ (ii) $x = -2$ (iii) $x = 0$

Answers