PROJECT MATHS

Text & Tests

Leaving 3 Certificate

chapter 17

Graphing Functions

Section 17.6 Using graphs of cubic functions

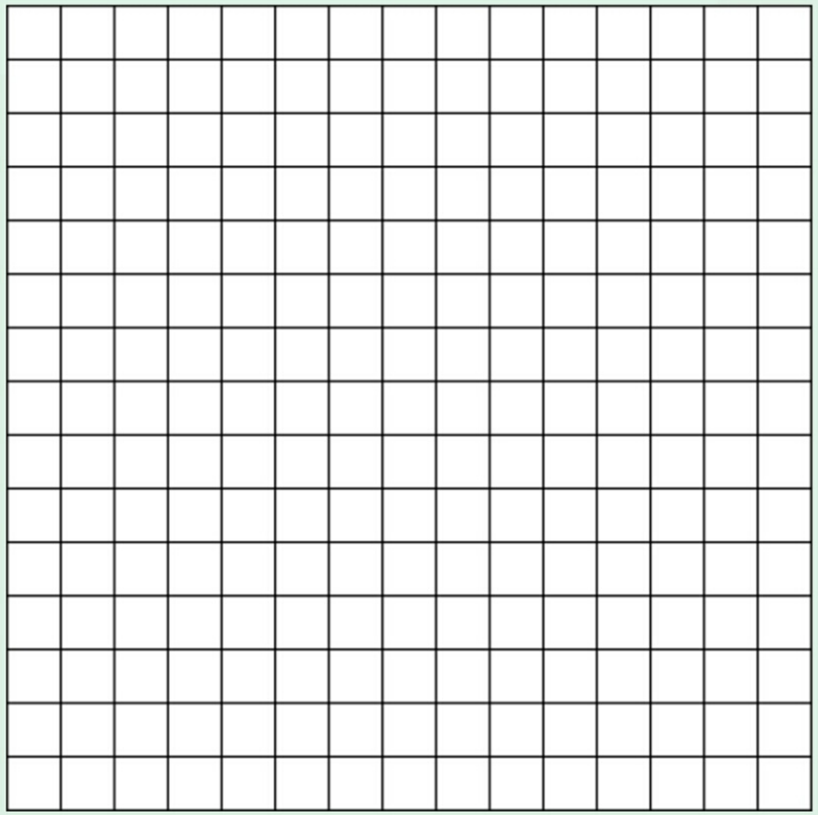
Example 1

Draw the graph of the function $y = x^3 - 3x^2 - 2x + 5$ in the domain $-2 \leqslant x \leqslant 4$.

Use your graph to solve the equations

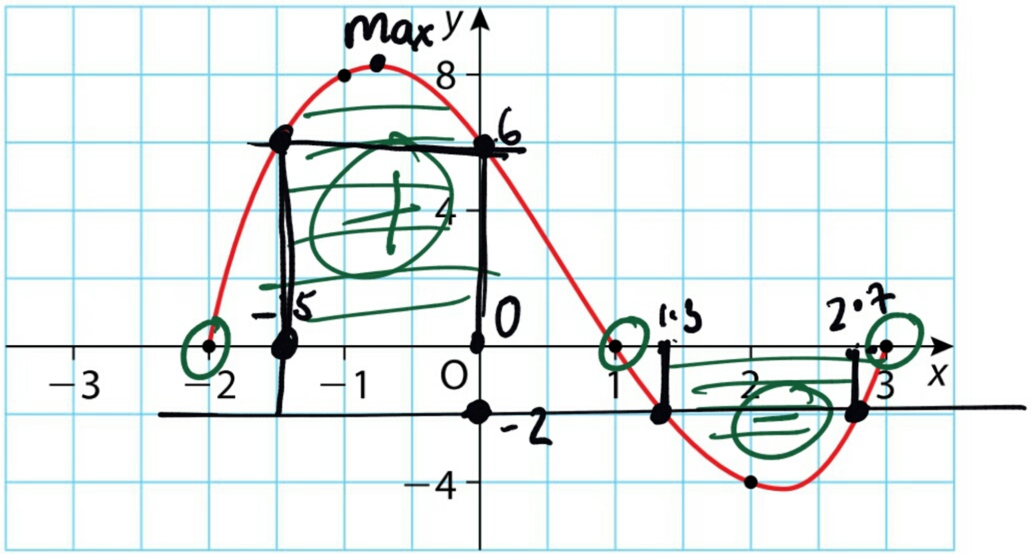
(i) $x^3 - 3x^2 - 2x + 5 = -3$

(ii) $x^3 - 3x^2 - 2x + 5 = 2x - 4$



Exercise 17.6

1. Drawn below is the graph of the function $f(x) = x^3 - 2x^2 - 5x + 6$ in the domain $-2 \leqslant x \leqslant 3, x \in R$.



Use your graph to write down, in the given domain,

(i) the roots of the equation $f(x) = 0$ cuts x axis $-2, 1, 3$.

(ii) the values of x for which $f(x) \geqslant 0$ Its positive $-2 \leq x \leq 1$

(iii) the coordinates of the maximum turning point $(-0.8, 8.1)$

(iv) the roots of the equation $x^3 - 2x^2 - 5x + 6 = -2$

(v) the roots of the equation $x^3 - 2x^2 - 5x = 0$. -1.5 and 0.

$f(x) = 6$

Exercise 17.6

2. Draw a graph of the function $f(x) = 2x^3 - x^2 - 8x + 4$ in the domain $-2 \leqslant x \leqslant 3$.
Find from your graph, as accurately as you can, the root(s) of the equations

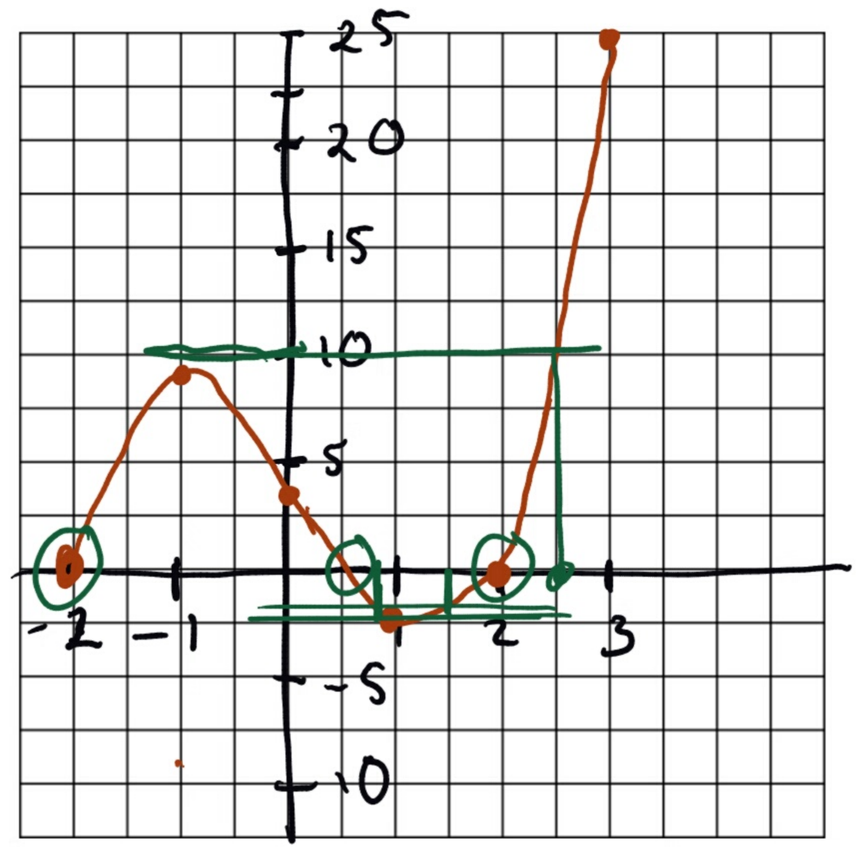
(i) $f(x) = 0$ $-2, .5, 2$

(ii) $2x^3 - x^2 - 8x + 4 = -3$ (1)

(iii) $2x^3 - x^2 - 8x - 6 = 0$. 2.5

$-6 + 10 = 4$.

x	$f(x)$
-2	0
-1	9
0	4
1	-3
2	0
3	25



Exercise 17.6

3. Here is the graph of the function

$$y = x^3 - x^2 - 8x + 12.$$

Use your graph to estimate

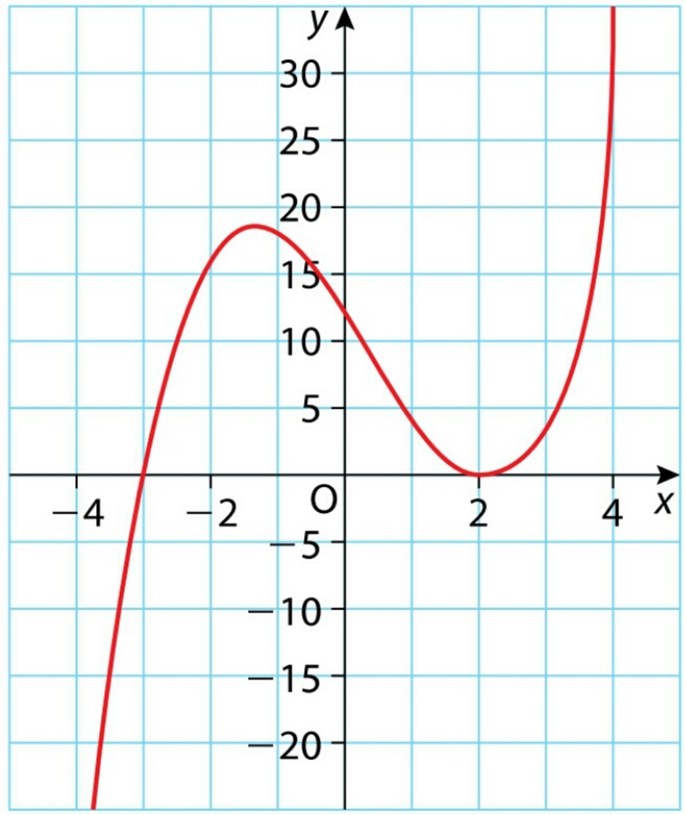
(i) the roots of the equation $y = 0$

(ii) the roots of the equation
$x^3 - x^2 - 8x + 12 = 5$

(iii) the values of x for which $y > 0$

(iv) the range of values of x for which y is positive and decreasing

(v) the repeated root of the equation
$x^3 - x^2 - 8x + 12 = 0$.



Exercise 17.6

4. Graph the function $f(x) = x^3 + 4x^2 + x - 6$ in the domain $-4 \leqslant x \leqslant 2, x \in R$.
Find from your graph as accurately as you can,

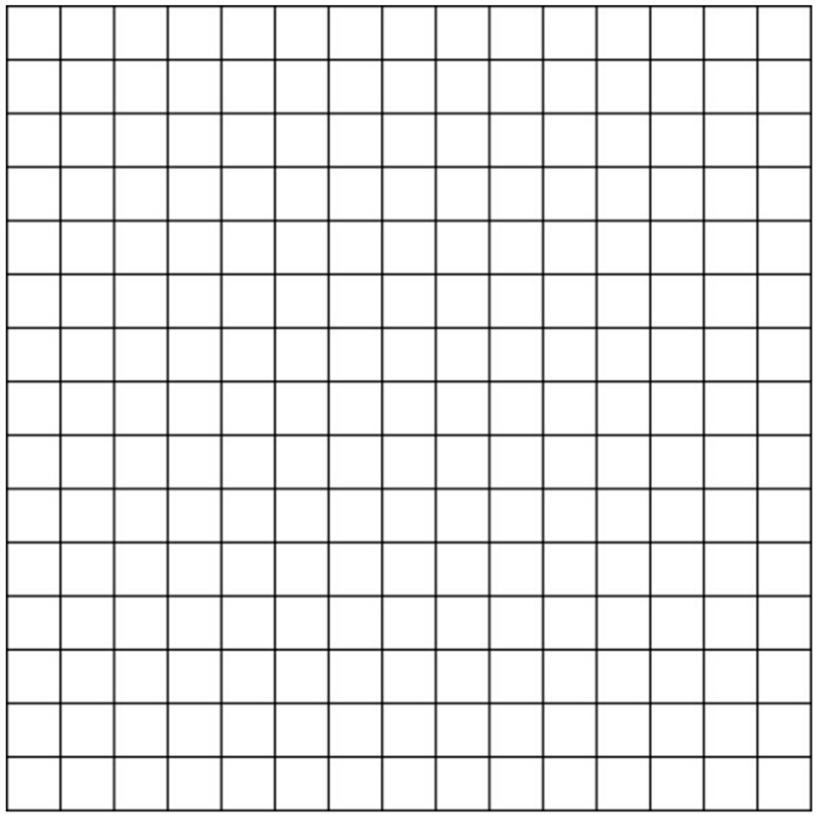
(i) the roots of the equation $f(x) = 0$

(ii) the real root of the equation $x^3 + 4x^2 + x - 9 = 0$

(iii) the values of x for which $f(x) < 0$

(iv) the values of x for which $f(x)$ is decreasing

(v) the roots of the equation $x^3 + 4x^2 + x - 6 = 2x + 4$.



Exercise 17.6

5. Here is the graph of the function

$$f(x) = x^3 - 3x^2 - 9x.$$

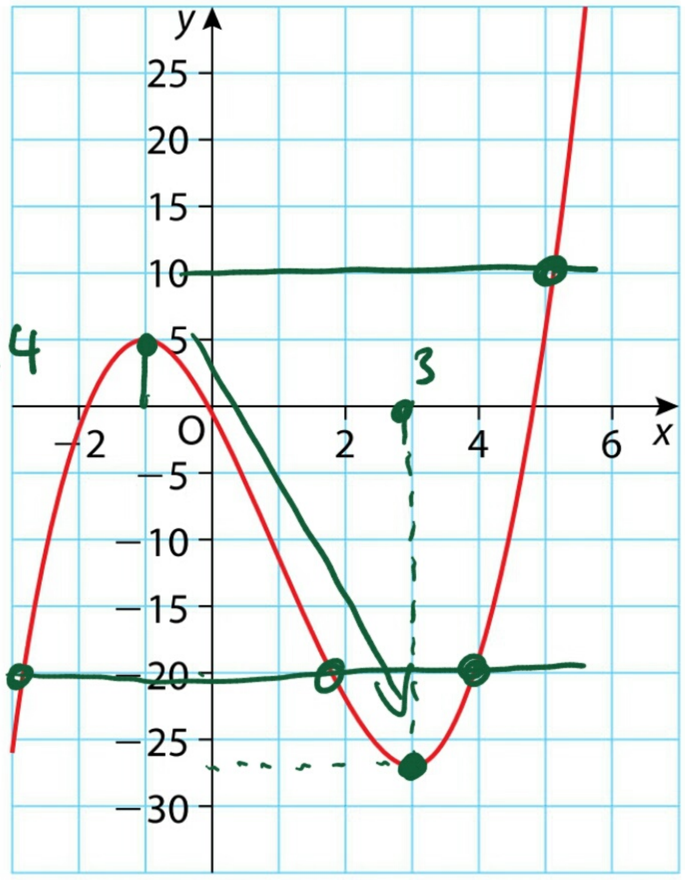
Use your graph to estimate

(i) $f(3)$ $x = -27$

(ii) the maximum turning point $(-1, 5)$ $-2.5, 1.8, 4$

(iii) the roots of the equation $x^3 - 3x^2 - 9x = -20$

(iv) the range of values of x for which $f(x)$ is decreasing. $-1 \leq x \leq 3$



Explain why the equation $f(x) = 10$ has only one root.

Explain why the equation $f(x) = -10$ has three roots.

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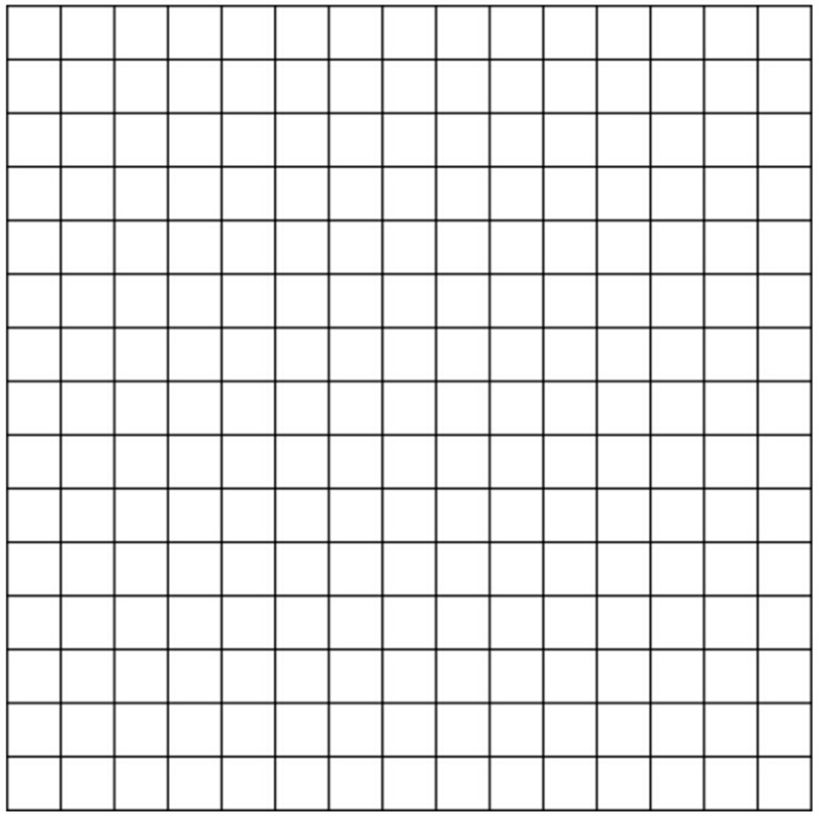
Exercise 17.6

6. Draw the graph of the function $f(x) = 3x - x^3$ in the domain $-2 \leqslant x \leqslant 2$.
Use your graph to solve these equations

(i) $-x^3 + 3x = 0$

(ii) $-x^3 + 3x + 1 = 0$

(iii) $-x^3 + 3x = x + 1$.



Exercise 17.6

7. A function $f(x)$ is of the form $f(x) = ax^3 + bx^2 + cx + d$, where a is positive.
Draw a rough sketch of $f(x)$ given that $y = f(x)$ satisfies these conditions:

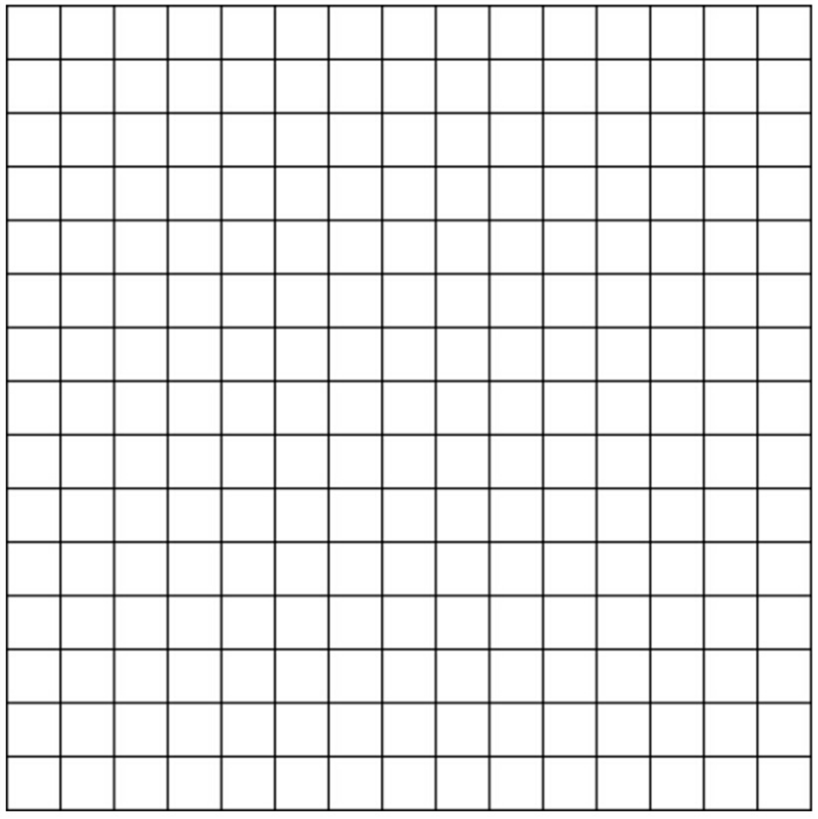
(a) $f(1) = 0$ and $f(9) = 0$

(b) $(1, 0)$ and $(5, -10)$ are turning points.

Use your graph to state

(i) the range of values of x for which $f(x) < 0$ when $x > 1$

(ii) the range of values of x for which $f(x)$ is negative and decreasing.



Exercise 17.6

8. Let $f(x) = 2x^3 - 5x^2 - 4x + 3$ for $x \in R$.
Copy and complete the table below:

x	-1.5	-1	0	1	2	3	3.5
$f(x)$	-9						13.5

Draw the graph of $f(x) = 2x^3 - 5x^2 - 4x + 3$ in the domain $-1.5 \leqslant x \leqslant 3.5$.
Use your graph to write down

(i) the coordinates of the minimum turning point

(ii) the roots of the equation $f(x) = 0$

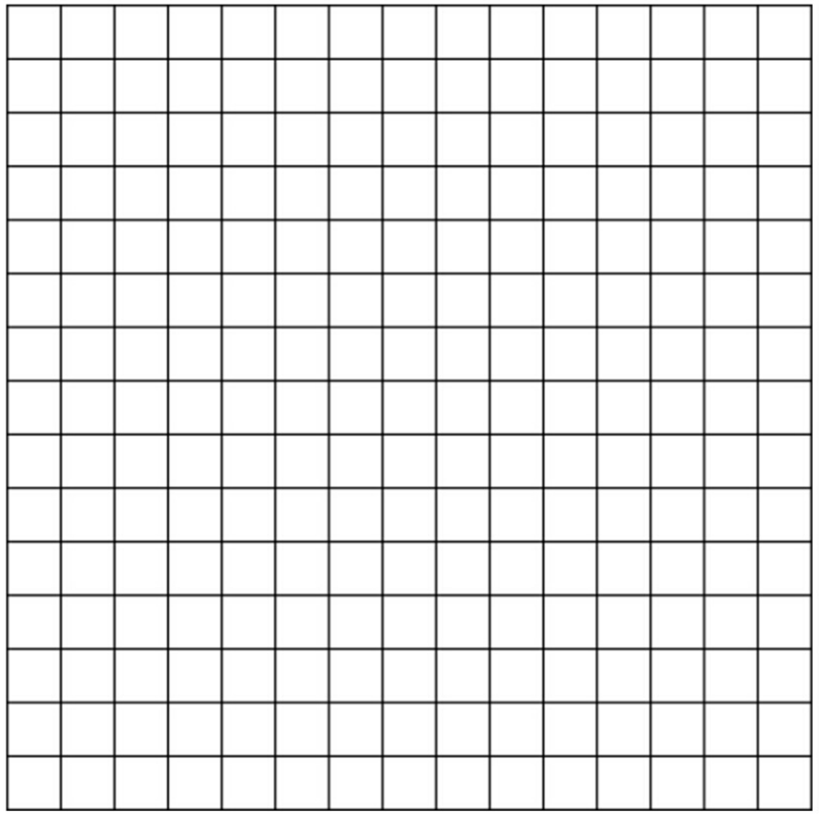
(iii) the roots of the equation $2x^3 - 5x^2 - 4x = 0$.

Write the equation $2x^3 - 5x^2 - 6x + 6 = 0$ in the form

$$2x^3 - 5x^2 - 4x + 3 = ax + b, \ a, b \in Z.$$

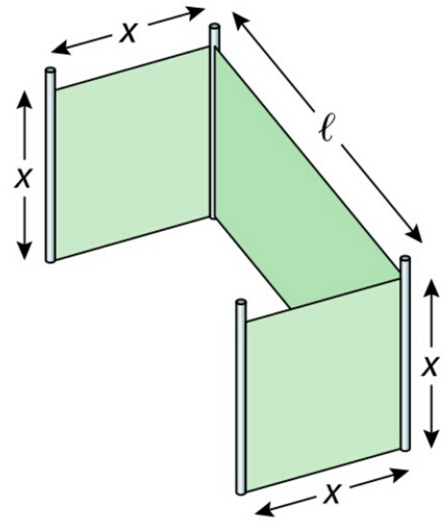
Hence use your graph to estimate the solutions of the equation

$$2x^3 - 5x^2 - 6x + 6 = 0.$$



Exercise 17.6

9. A canvas wind-shelter has two square ends, each of side x metres, and a rectangular back of length ℓ metres. The area of canvas is 9 m^2.



(i) Show that $\ell = \dfrac{9}{x} - 2x$, and hence show that the enclosed volume, V m^3, is $9x - 2x^3$.

(ii) Plot the graph of V against x for $0 \leqslant x \leqslant 3$.

(iii) (a) Use your graph to find the value of x that gives the largest possible volume.

(b) From your graph, what is this largest volume?

Answer 17.6

1. (i) $-2, 1, 3$ (ii) $-2 < x < 1$
(iii) $(-0.8, 8.2)$ (iv) $1.4, 2.8$
(v) $-1.5, 0$

2. (i) $-1.35, -0.6, 2.45$
(ii) $-1.7, -0.1, 2.3$
(iii) 2.55

3. (i) $-3, 2$
(ii) $-2.8, 0.85, 2.9$
(iii) $-3 < x < 2; 2 < x < 4$
(iv) $-1\frac{1}{3} < x < 2$ (v) $x = 2$

4. (i) $-3, -2, 1$ (ii) 1.2
(iii) $-2 < x < 1$ (iv) $-2.5 < x < 0$
(v) $-3.45, -2, 1.45$

5. (i) -27 (ii) $(-1, 5)$
(iii) $-2.8, 1.8, 4$
(iv) $-1 < x < 3$; one point of intersection of curve and line $y = 10$; three points of intersection of curve and line $y = -10$

6. (i) $-1.7, 0, 1.7$ (ii) $-1.5, -0.35, 1.9$
(iii) $-1.6, 0.6, 1$

7. (i) $-1 < x < 9$ (ii) $1 < x < 5$

8. (i) $(2, -9)$ (ii) $-1, 0.5, 3$
(iii) $-0.65, 0, 3.15$;
$2x^3 - 5x^2 - 4x + 3 = 2x - 3$;
$-1.35, 0.7, 3.15$

9. (iii) (a) 1.2 m (b) 7.35 m^3