PROJECT MATHS

Text & Tests 3

Leaving Certificate

chapter 17

Graphing Functions

Section 17.4 Quadratic graphs and real-life problems —

Example 1

Given that $f(x) = 4 - 3x - x^2, x \in R$, copy and complete the given table.

Draw the graph of $f(x)$ in the domain $-5 \leqslant x \leqslant 2$.

If the graph represents the temperature, in °C, taken every two hours between 6 a.m. $(x = -5)$ and 8 p.m. $(x = 2)$ in a certain city, use the graph to estimate

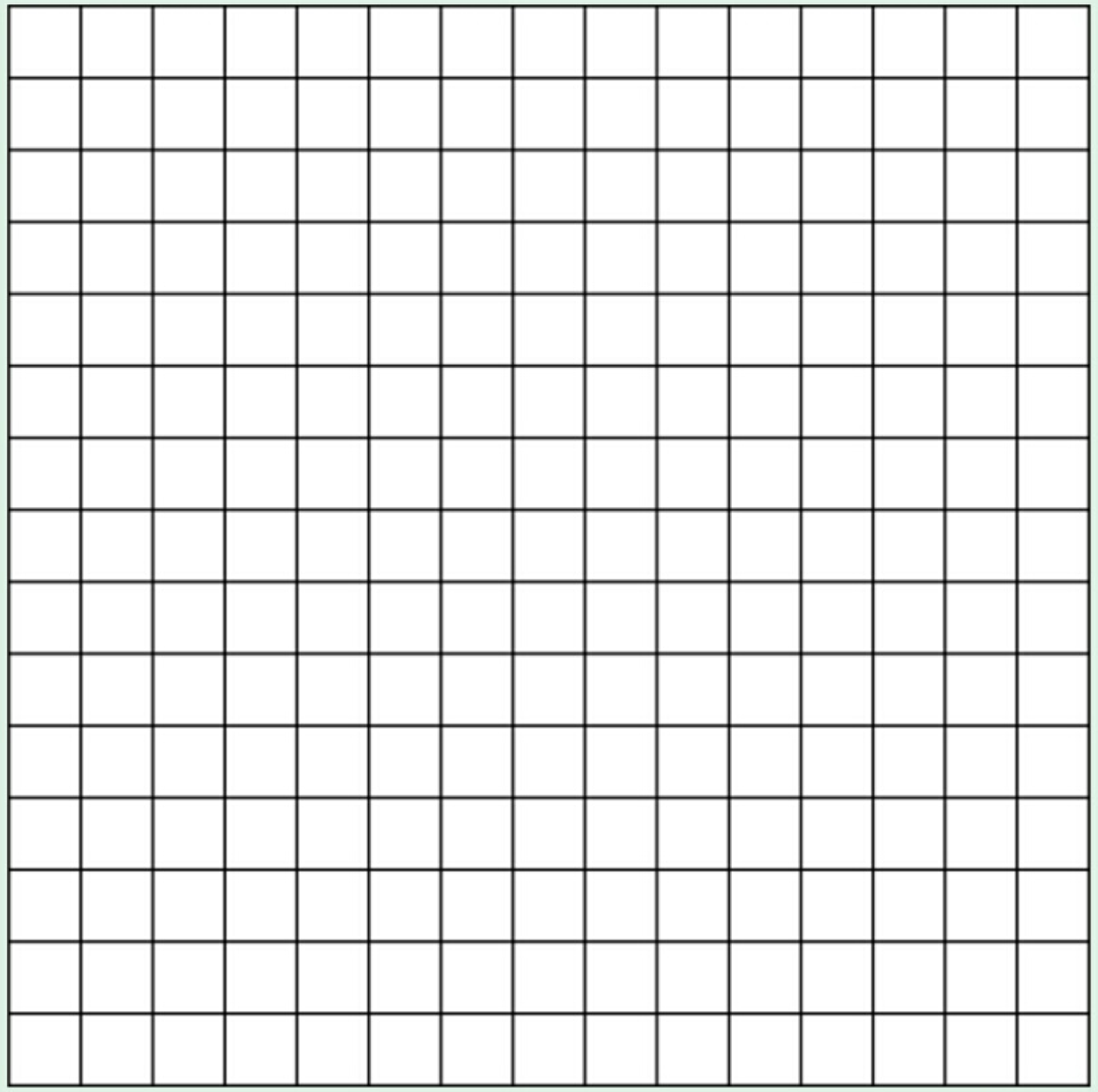
(i) the temperature at 11 a.m.

(ii) the time when the temperature was highest

(iii) the times when the temperature was 3°C

(iv) the number of hours the temperature was at or above freezing point.

x	$4 - 3x - x^2$	y
-5	$4 + 15 - 25$	-6
-4		
-3		
-2		
-1		
0		
1	$4 - 3 - 1$	0
2		



Exercise 17.4

1. On the right is the graph of the function

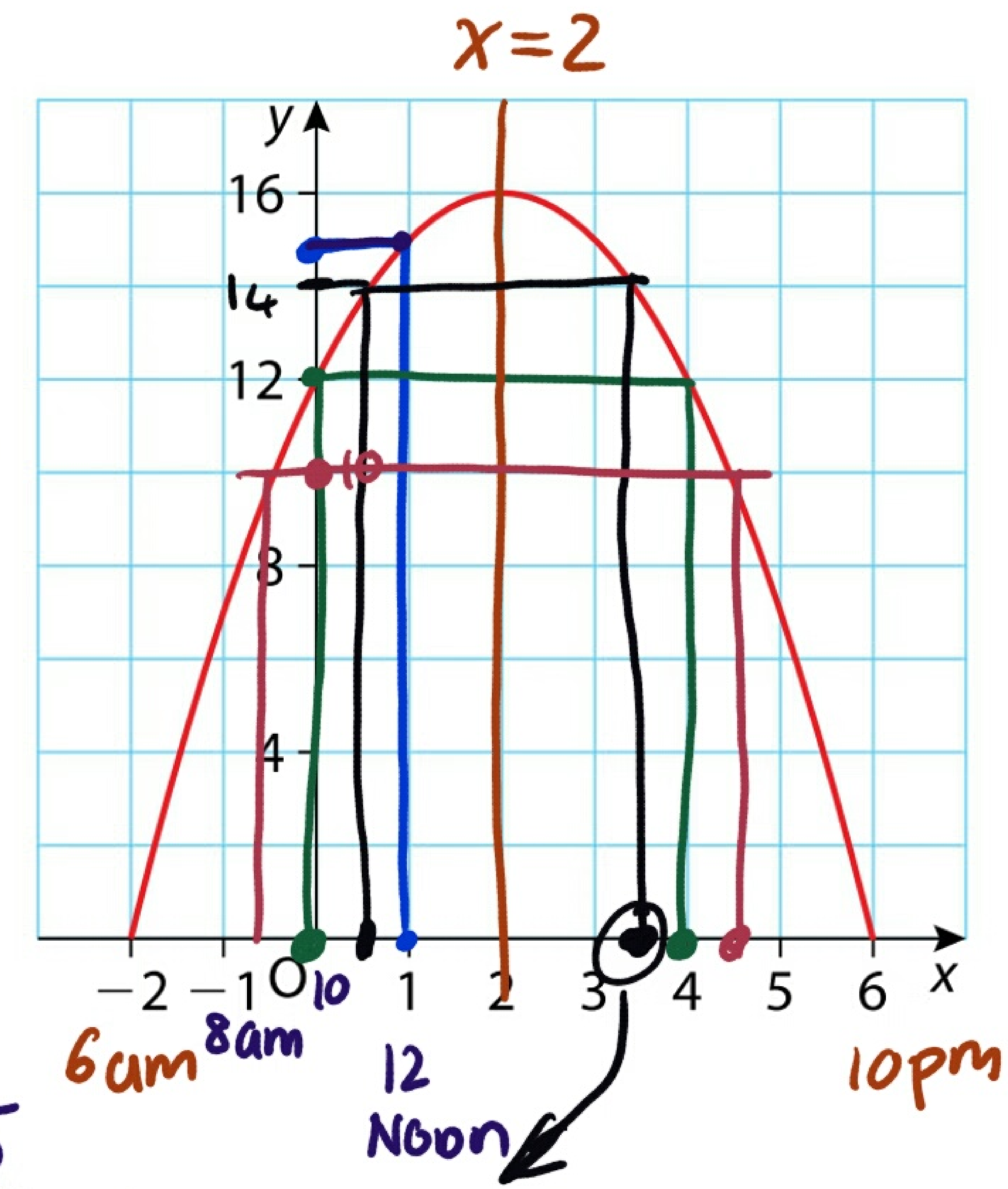
$f(x) = -x^2 + 4x + 12.$

Use the graph to write down

(i) $f(1)$ = 15

(ii) the values of x for which $f(x) = 12$ (0 and 4)

(iii) the equation of the axis of symmetry.



$f(x)$ represents the number of taxis at a taxi-rank from 6 a.m. $(x = -2)$ to 10 p.m. $(x = 6)$.

Each unit on the x-axis represents 2 hours and each unit on the y-axis represents one taxi.

Use the graph to estimate

(iv) the number of taxis at the rank at 12 noon ≈ 15

(v) the times when there were 14 taxis at the rank ≈ 11am and 5pm

(vi) the number of hours during which there were 10 taxis or more at the rank. 9am 7pm.

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Exercise 17.4

2. Graphed on the right is the function

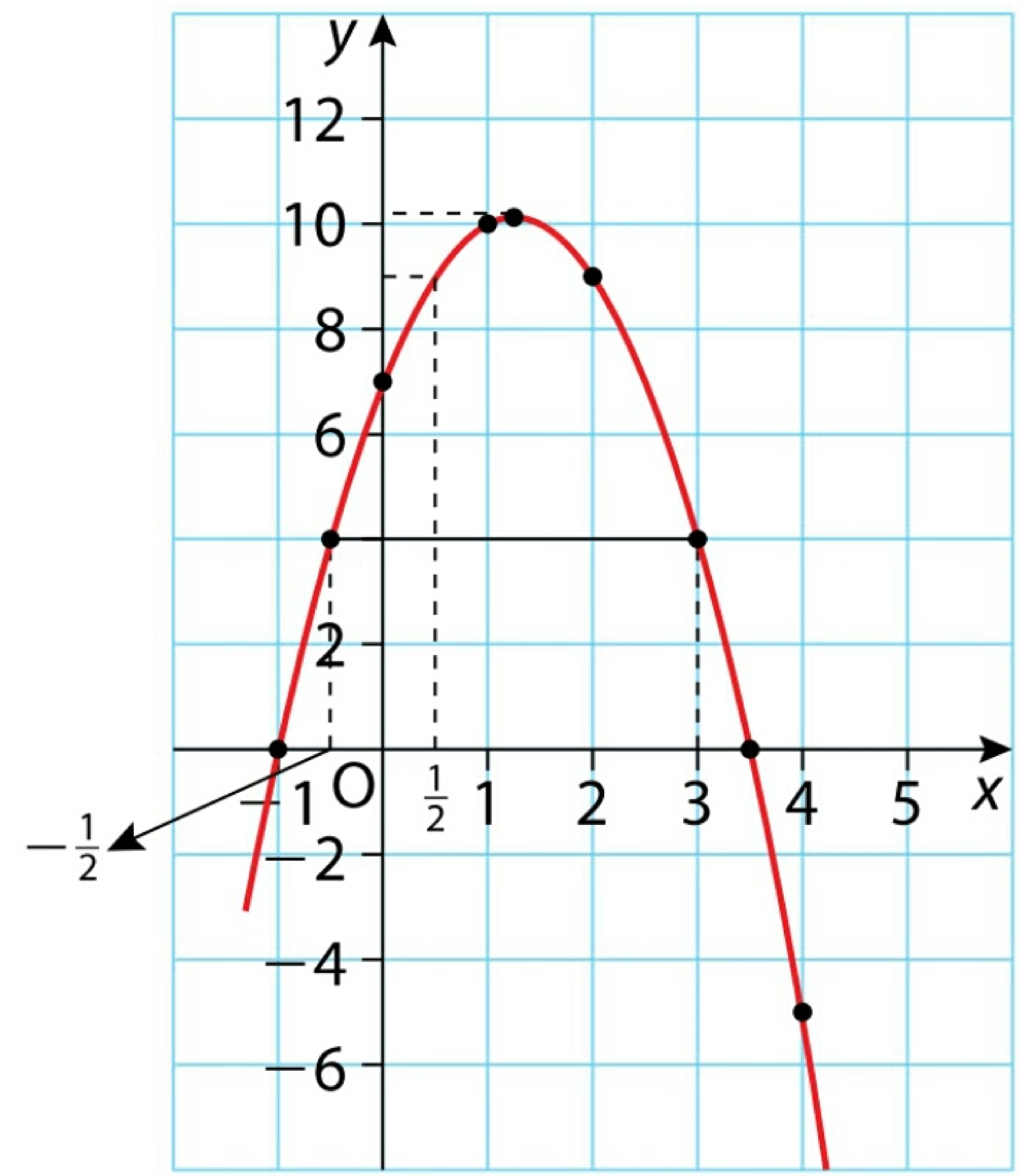
$$f(x) = 7 + 5x - 2x^2$$

in the domain $-1 \leqslant x \leqslant 4$.

Use your graph to solve

$$7 + 5x - 2x^2 = 0.$$

$f(x)$ is the height, in metres, reached by a particle fired from level ground at the point where $x = -1$, the x-axis representing level ground. From the time of firing until it hits the ground again, the particle was in flight for exactly 4.5 seconds.



Use your graph to estimate

(i) the maximum height reached by the particle

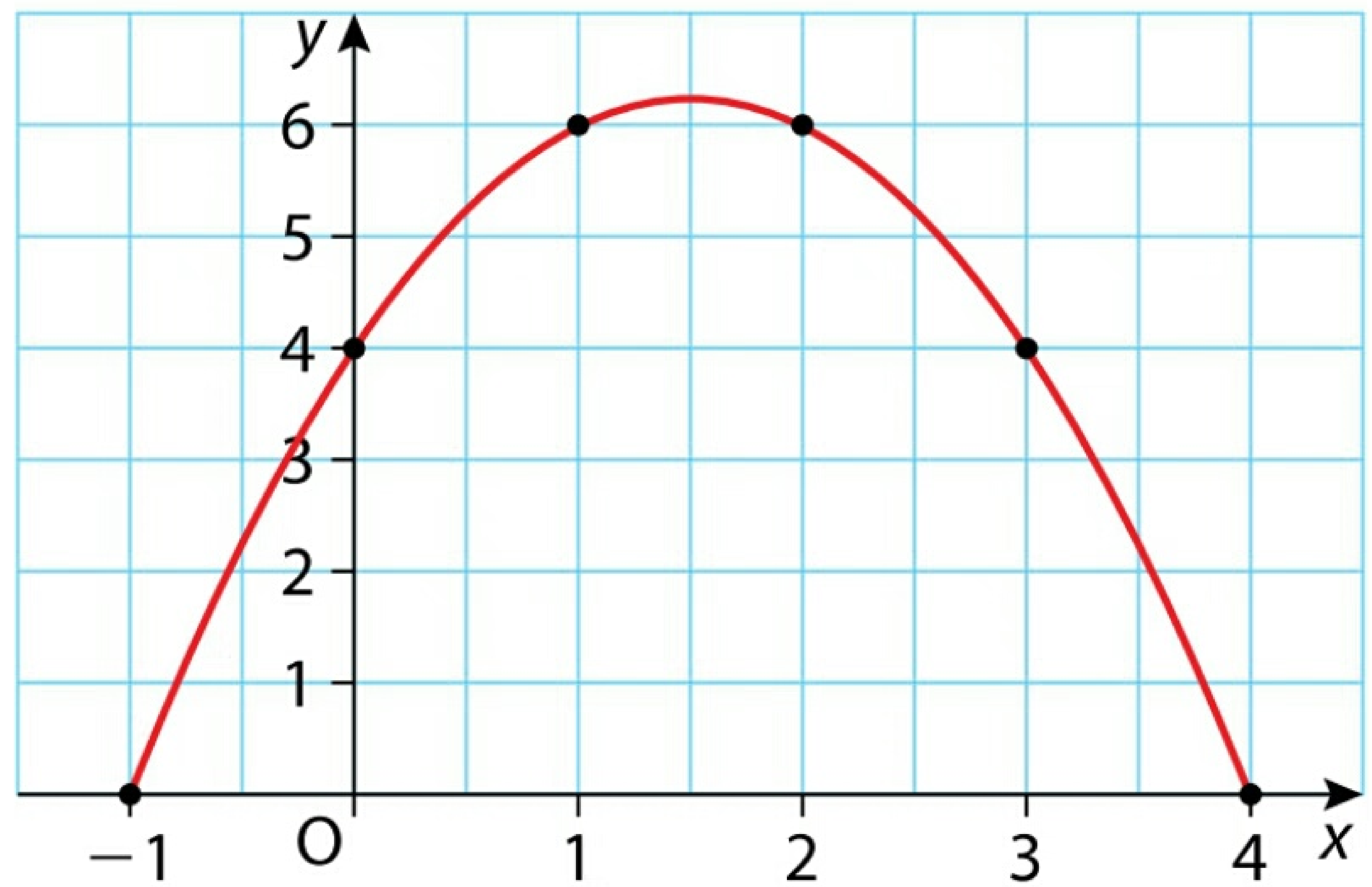
(ii) the height reached by the particle after 1.5 seconds of flight

(iii) the number of seconds the particle is 4 m or more above the ground.

Exercise 17.4

3. Sketched below is the graph of $f(x) = 4 + 3x - x^2$ in the domain $-1 \leqslant x \leqslant 4$.

H/w



The graph represents the number of cars in a car park between 9 a.m. and 12 midnight.
Each unit on the y-axis represents 100 cars.
Each unit on the x-axis represents 3 hours, where
$-1 = 9$ a.m., $0 = 12$ noon, $1 = 3$ p.m., … etc.

Use your graph to estimate

(i) the number of cars in the car park at 1.30 p.m.

(ii) the times when the car park contained 400 cars

(iii) the time the car park contained the greatest number of cars and write down this number

(iv) the times that the car park contained no cars.

Exercise 17.4

4. A farmer has 16 metres of fencing with which to make a rectangular enclosure for sheep. If one side of the enclosure is x metres long, show that the area A is given by $A(x) = 8x - x^2$.

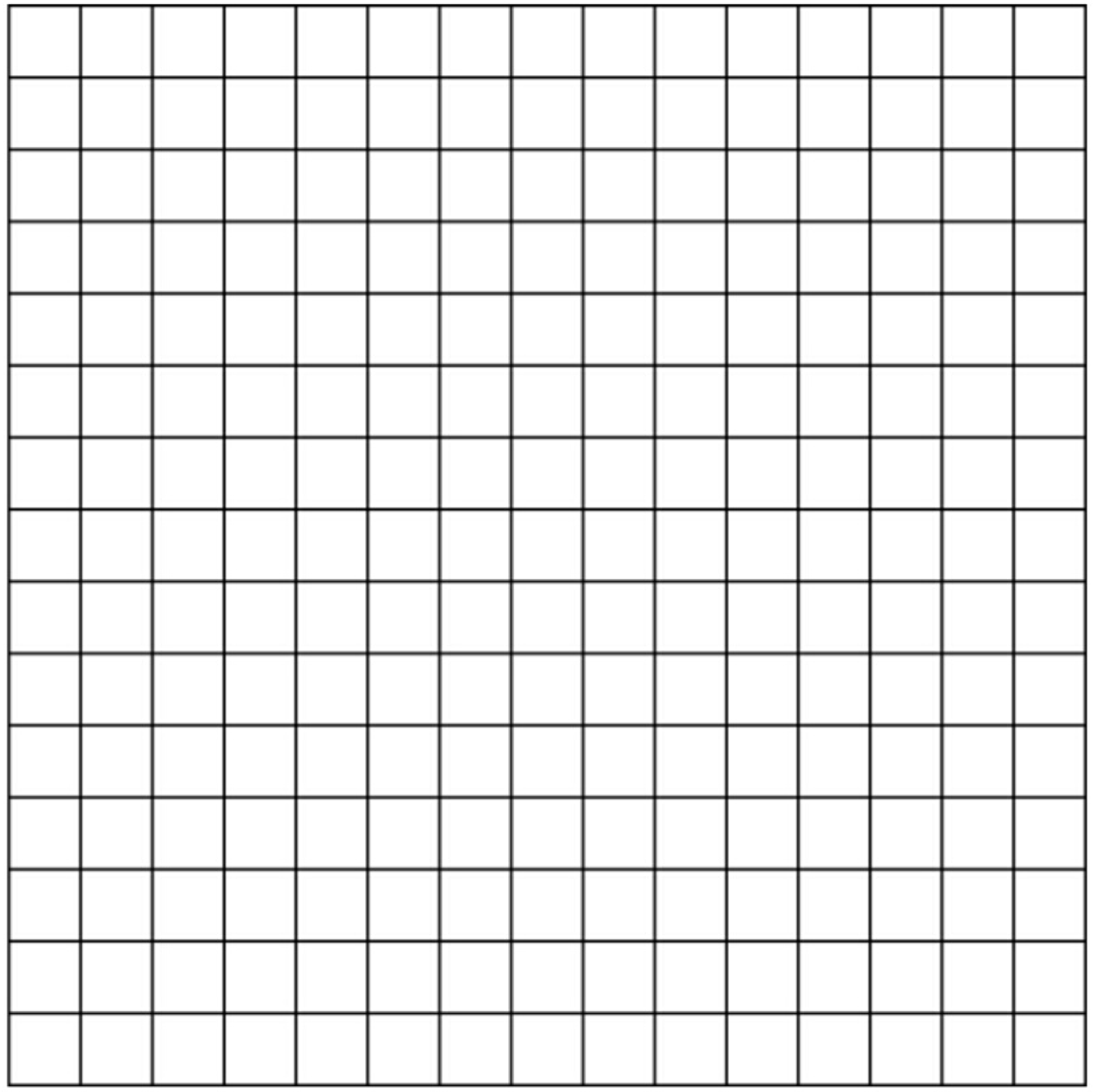
Draw the graph of $A(x)$ in the domain $0 \leqslant x \leqslant 8$.

Use your graph to estimate

(i) the area of the enclosure when $x = 2.5$

(ii) the maximum possible area and the value of x when this occurs

(iii) the two values of x for which the area is $12\ \text{m}^2$.



Exercise 17.4

5. Draw the graph of the function $f: x \rightarrow 6x - x^2$ in the domain $0 \leqslant x \leqslant 6$.

$f(x)$ represents the height, in metres, reached by a golf ball from the time it was hit $(x = 0)$ to the time it hit the ground $(x = 6)$.

If each unit on the x-axis represents 1 second and each unit on the y-axis represents 5 metres, use your graph to estimate

(i) the greatest height reached by the golf ball

(ii) the height of the golf ball after $1\frac{1}{2}$ seconds

(iii) after how many seconds the ball was 10 metres above ground

(iv) after how many seconds the ball reached its maximum height.

Exercise 17.4

6. The area of a circle is given roughly by the formula $A = 3r^2$.

(i) Copy and complete the table given on the right and draw a graph of the function for $0 \leqslant r \leqslant 3$.

(ii) Use your graph to find an estimate for the area of a circle of radius 2.5 m.

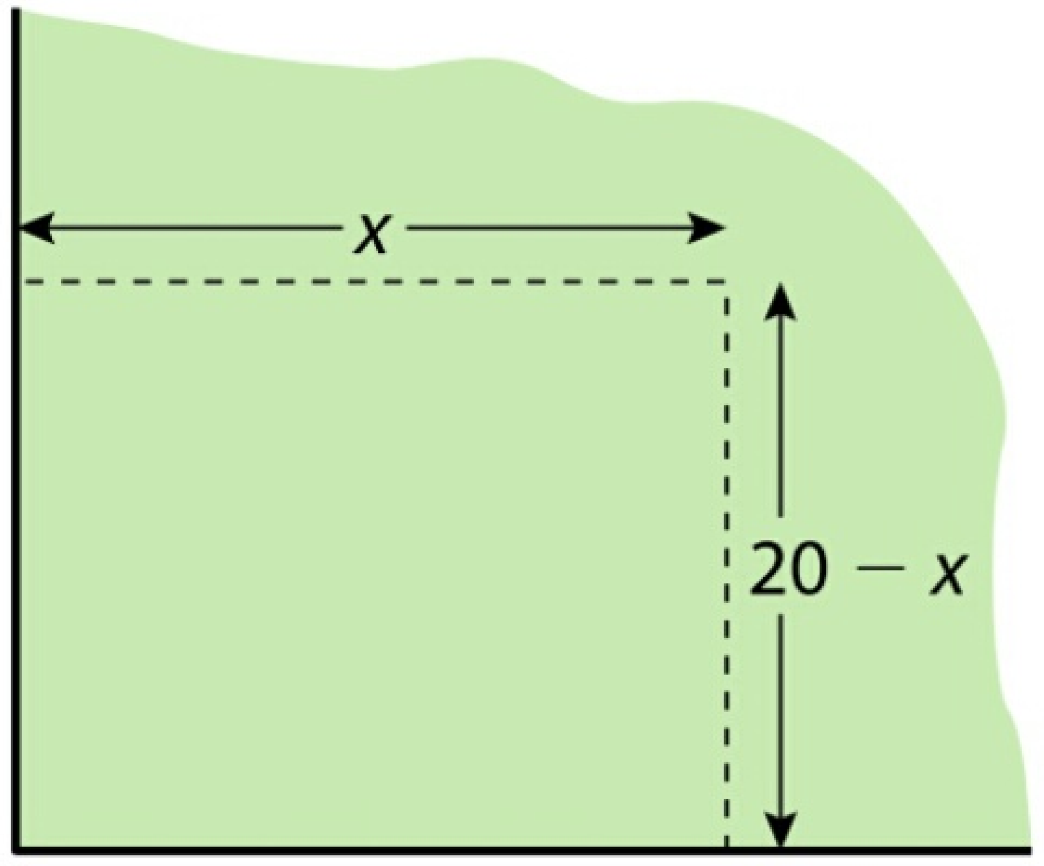
(iii) If a circle has an area of 10 m^2, use the graph to estimate the length of its radius.

(iv) Check your answers to parts (ii) and (iii) using the accurate version of the formula for the area of a circle (i.e. $A = \pi r^2$).

r	$3r^2$	A
0		
1	3	3
2		
3		

Exercise 17.4

7. A farmer has 20 metres of fencing.
He wishes to use it to form a rectangular enclosure in the corner of a field, as in the diagram.



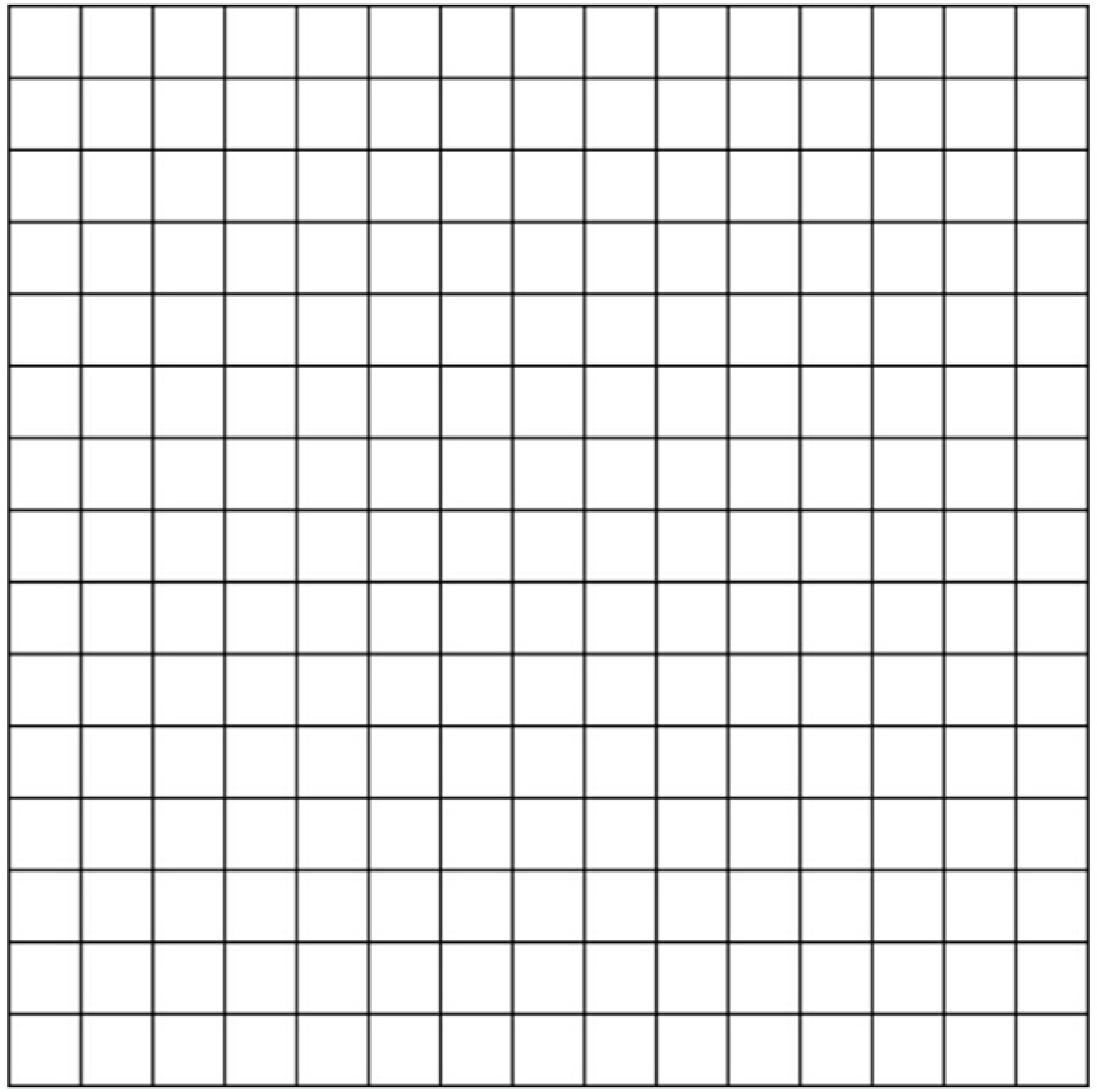
(i) Write down an expression for the area, A m^2, enclosed by the fencing.

(ii) Plot the graph of A for values of x between 0 and 20.

(iii) For what values of x is the area 40 m^2?

(iv) What range of values of x give an enclosed area greater than 90 m^2?

(v) What is the maximum area the farmer can enclose?
What are the lengths of the fencing for this maximum area?



Answers 17.4

1. (i) 15 (ii) 0, 4
(iii) $x = 2$ (iv) 15 taxis
(v) 11 a.m., 5 p.m. (vi) 10 hours

2. $x = -1, 3.5$;
(i) 10.1 m (ii) 9 m (iii) 3.5 secs

3. (i) 525
(ii) 12.00 midday, 9 p.m.
(iii) 4.30 p.m.; 625
(iv) 9 a.m., 12 midnight

4. (i) 13.75 (ii) 16 m^2; $x = 4$
(iii) 2, 6

5. (i) 45 m (ii) 34 m
(iii) 0.4, 5.7 secs (iv) 3 secs

6. (i) (0, 0), (1, 3), (2, 12), (3, 27)
(ii) 19 m^2
(iii) 1.8 m
(iv) 19.6 m^2; 1.78 m

7. (i) $(20x - x^2)$ m^2
(iii) 17.75, 2.25
(iv) $6.8 < x < 13.2$
(v) 100 m^2; 10 m by 10 m