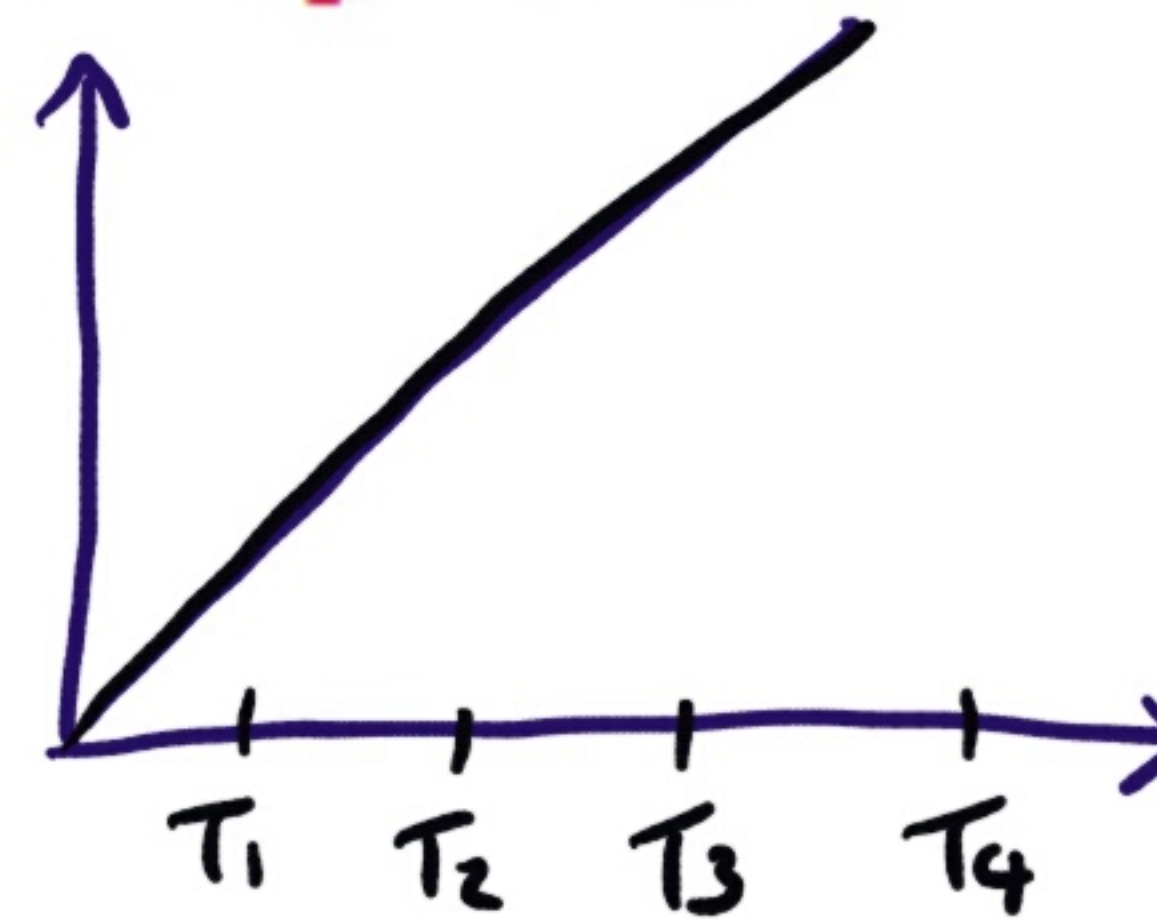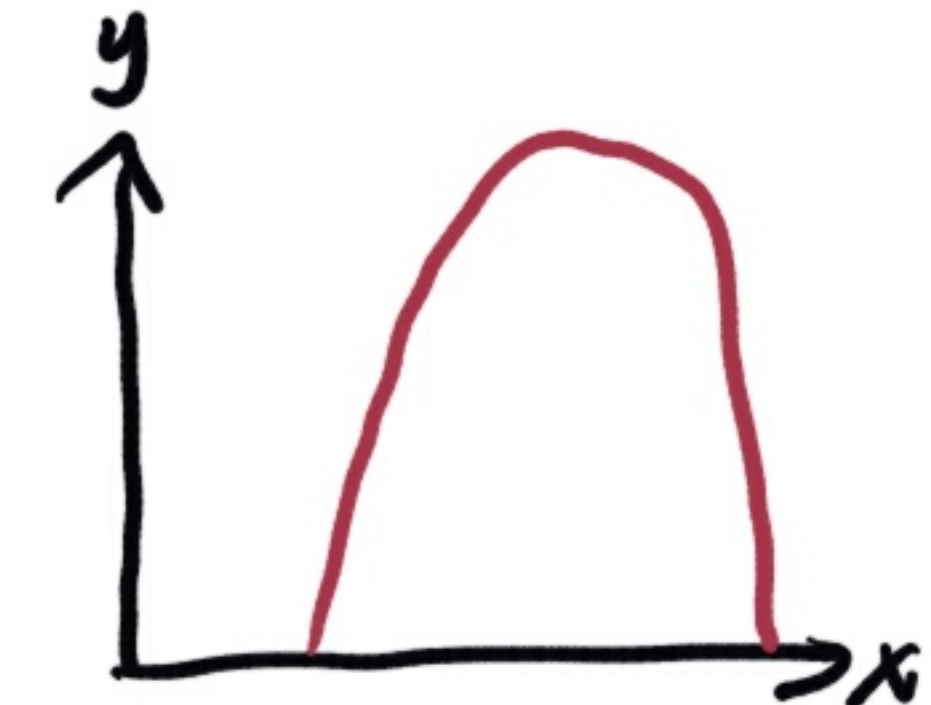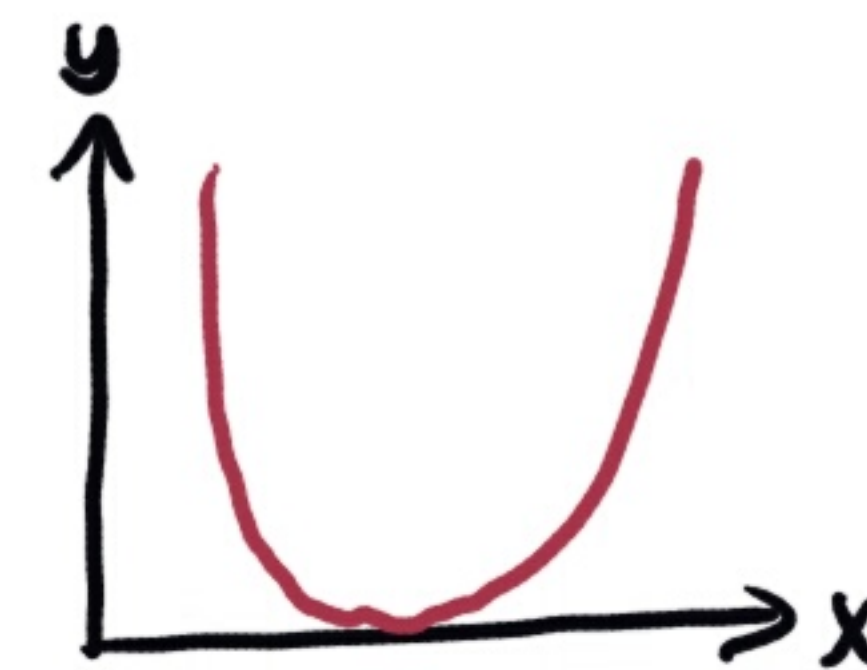# Chapter 18 — Patterns and Sequences

Section 18.7 Graphing sequences

372

① Linear/Arithmetic

$T_n = an + b$

② Quadratic sequences

$T_n = an^2 + bn + c$

③ Exponential sequences

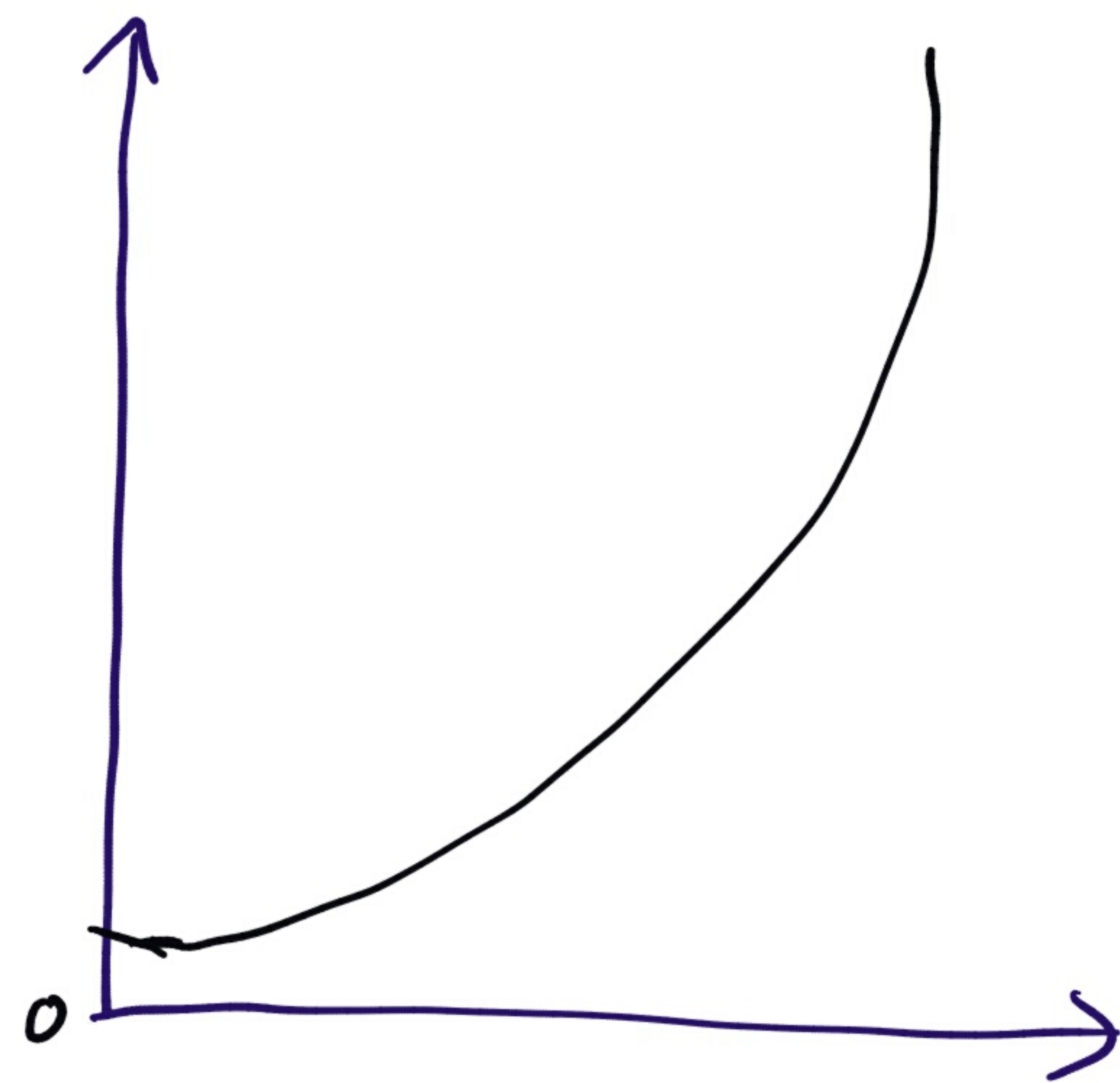The power is the variable

① $2^0, 2^1, 2^2, 2^3, 2^4 \implies 2^n$

$1, 2, 4, 8, 16 \ldots$

② $3^0, 3^1, 3^2, 3^3, 3^4, 3^5 \implies 3^n$

$1, 3, 9, 27, 81, 243$

Population / Bacteria
Growth

Example 1

Here are the fare structures of two taxi companies.

Company A has a fixed charge of €4 plus €2 per kilometre
Company B has no fixed charge but charges €2.50 per kilometre travelled.

Draw a graph to represent these two companies' charges.
Put the charges on the *x*-axis and the distances on the *y*-axis.

Example 1

Use your graph to find

(i) how much Company A charges for a journey of 10 km

(ii) how far you could travel with Company B for €16

(iii) the distance for which both companies charge the same amount

(iv) the difference in the charges when the distance is 4 km.

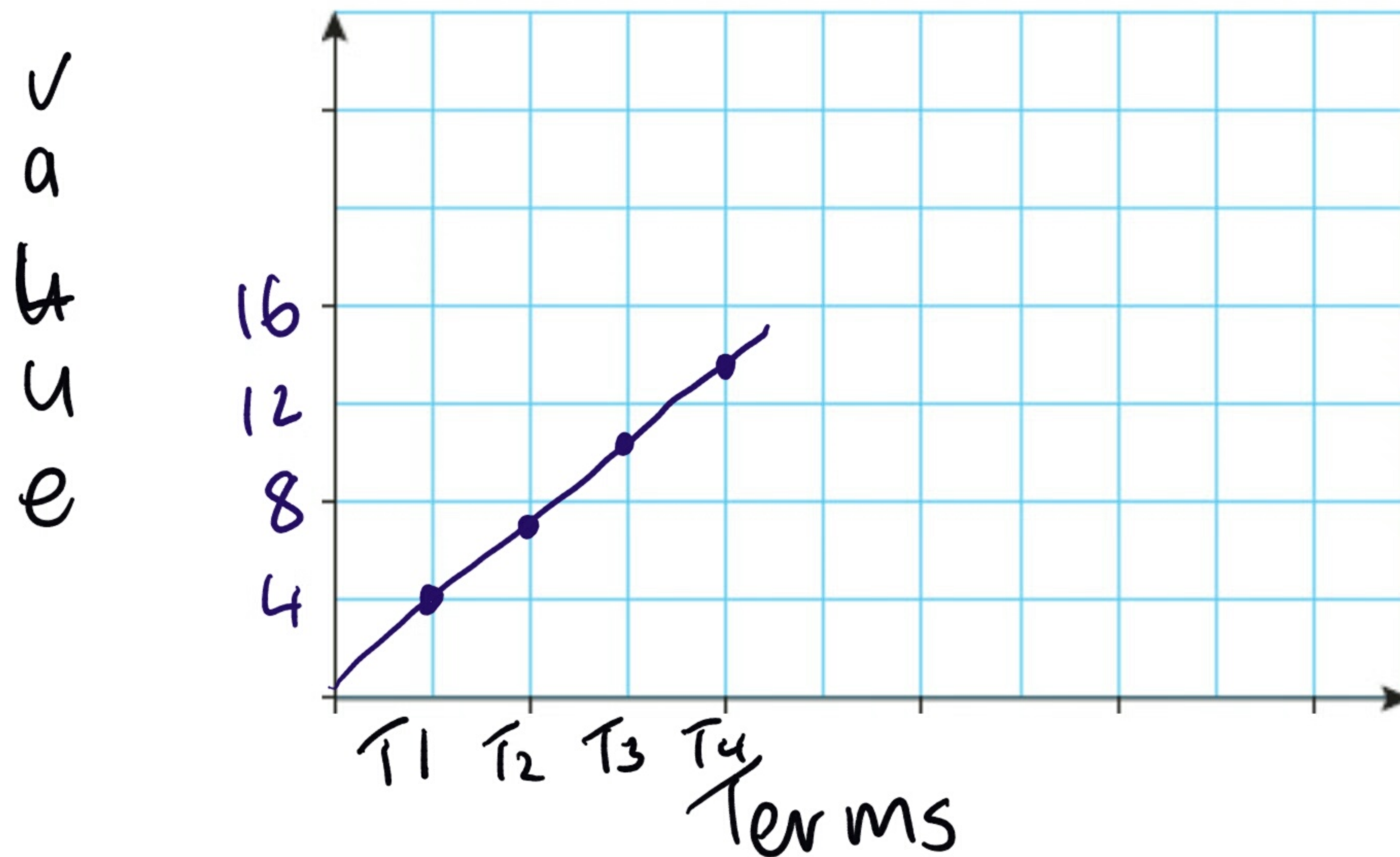**1.** The table on the right shows the first four terms of a sequence and the values of these terms.

Term	1	2	3	4
Value	4	7	10	13

→3 →3 →3   first diff

constant.

Putting the terms on the horizontal axis and the values on the vertical axis, draw a graph of the sequence.

Explain why the sequence is linear.

going up in
a straight
line.

v
a
l
u
e

16
12
8
4

T1 T2 T3 T4
Terms

2. If $T_n = 2n + 1$, write out the first five terms of the sequence.

Illustrate this sequence on a graph, putting the term numbers on the horizontal axis.

Explain why the graph is a straight line.

linear sequence

5, 7, 9, 11   first diff constant

$T_1 = 2(1)+1$
$2+1$
$T_1 = 3$

Term

$T_2 = 2(2)+1$
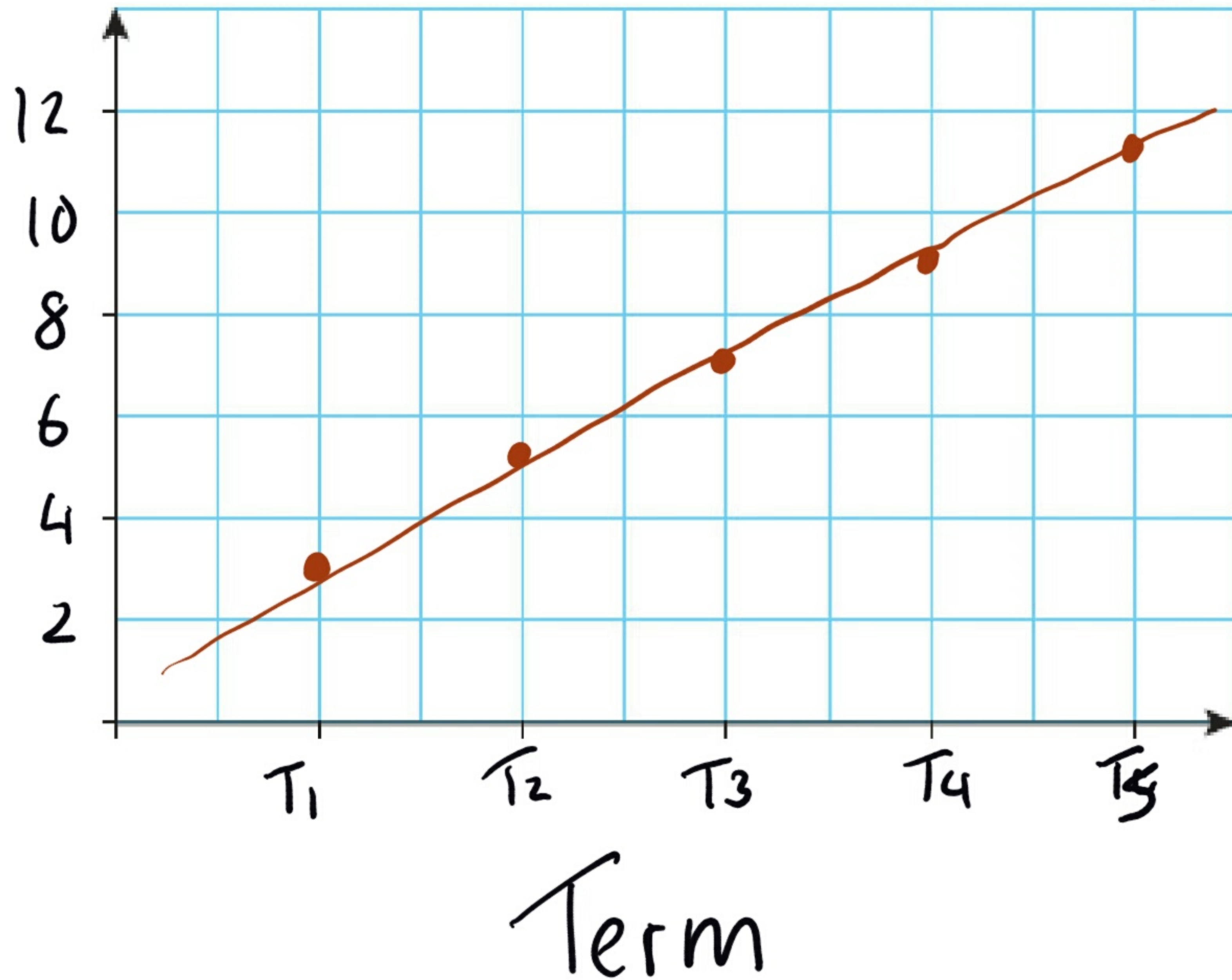$4+1$
$T_2 = 5$

$T_3 = 2(3)+1$
$6+1$
$T_3 = 7$

$T_4 = 2(4)+1$
$8+1$
$T_4 = 9$

$T_5 = 2(5)+1$
$10+1$
$T_5 = 11$

HW
Pg 375

3. The graph on the right shows a plumber's charges when called to do a repair job on a boiler.

(i) What is the initial or 'call out' charge? *€30*

(ii) How much does he charge for a job that lasts for $3\frac{1}{2}$ hours? *€105*

(iii) If he charges €135, how many hours has he worked? *5 hours*

(iv) Do his charges form a linear sequence? *Yes.* Explain your conclusion.

(v) Use the graph to work out what the plumber charges for each hour of actual work (i.e. excluding the 'call out' charge). *$135 - 30 = 105 \div 5 = €21$*

(vi) Investigate if the slope of the line is the same as the rate he charges for each hour's work. *both 21*

(vii) If the work lasted 10 hours, use the sequence to work out what the charge would be. *$21 \times 10 = 210 + 30 = €240$*

Charge (in €)

*(135*

150
120
90
60
30

O   1   2   3   4   5
Time (in hours)
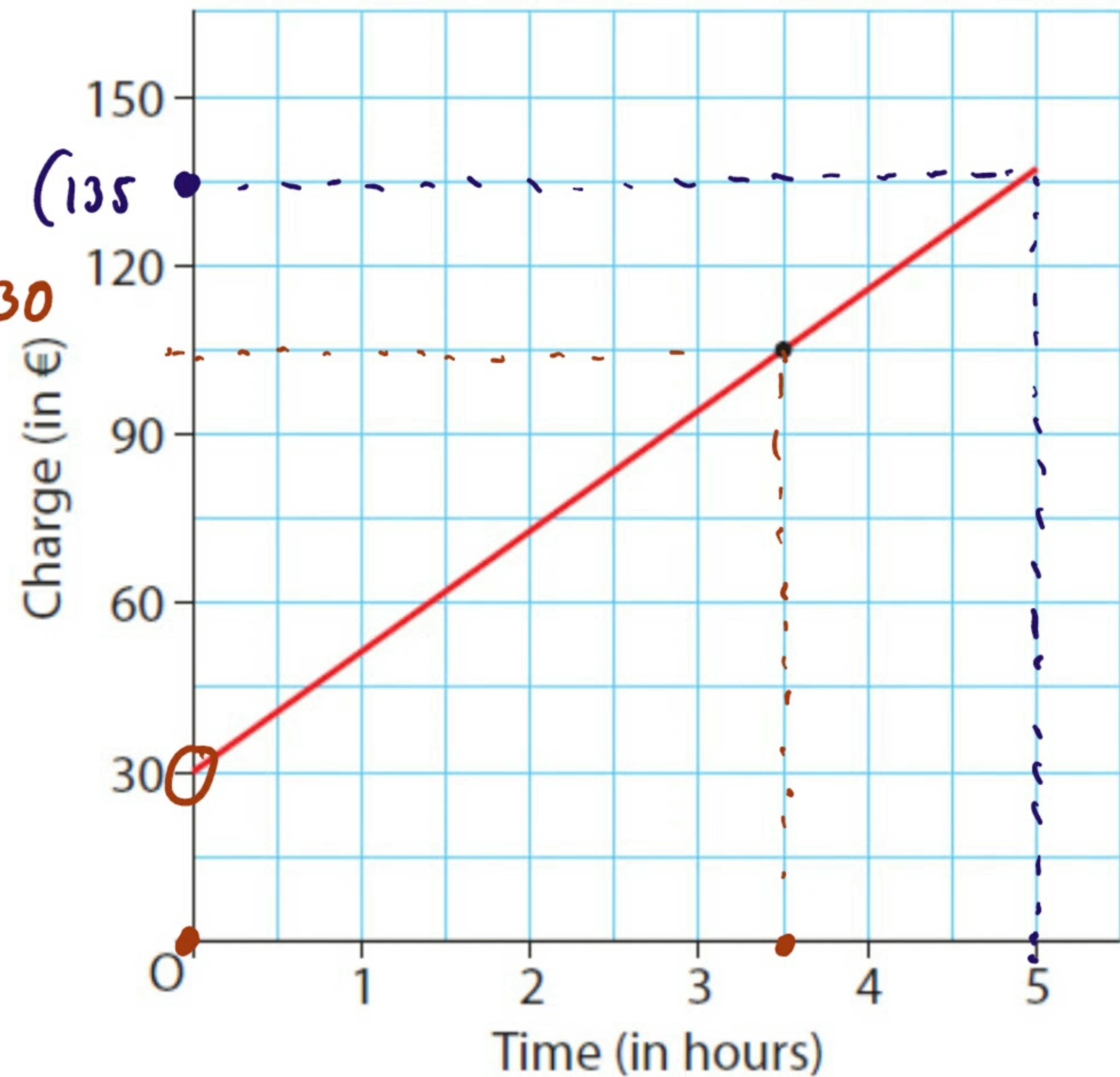
4. The table on the right shows the total amounts of money Cara has saved after weeks 1, 2, 3, ….

Draw a graph to illustrate these savings, putting the week number on the x-axis.

Week number	Amount saved
1	€10
2	€20
3	€30
4	€40
5	€50
……	……

(i) Is the graph linear?

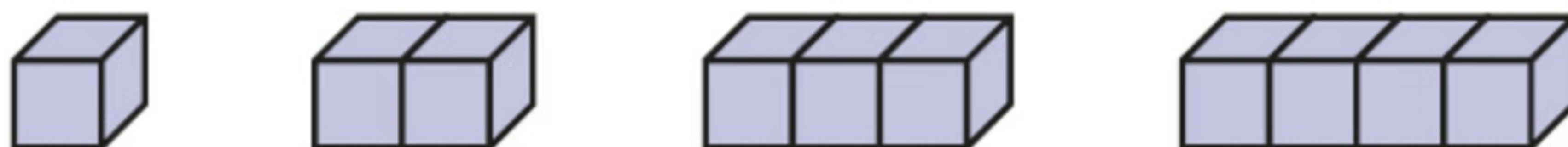(ii) Use the pattern to find the amount she will have saved at the end of the 12th week.

~~(iii) Find the equation of the line you have drawn.~~

~~(iv) Use the equation to find what Cara will have saved at the end of the 20th week.~~

y

week number

M
O
N
E
Y

S
A
V
E
D
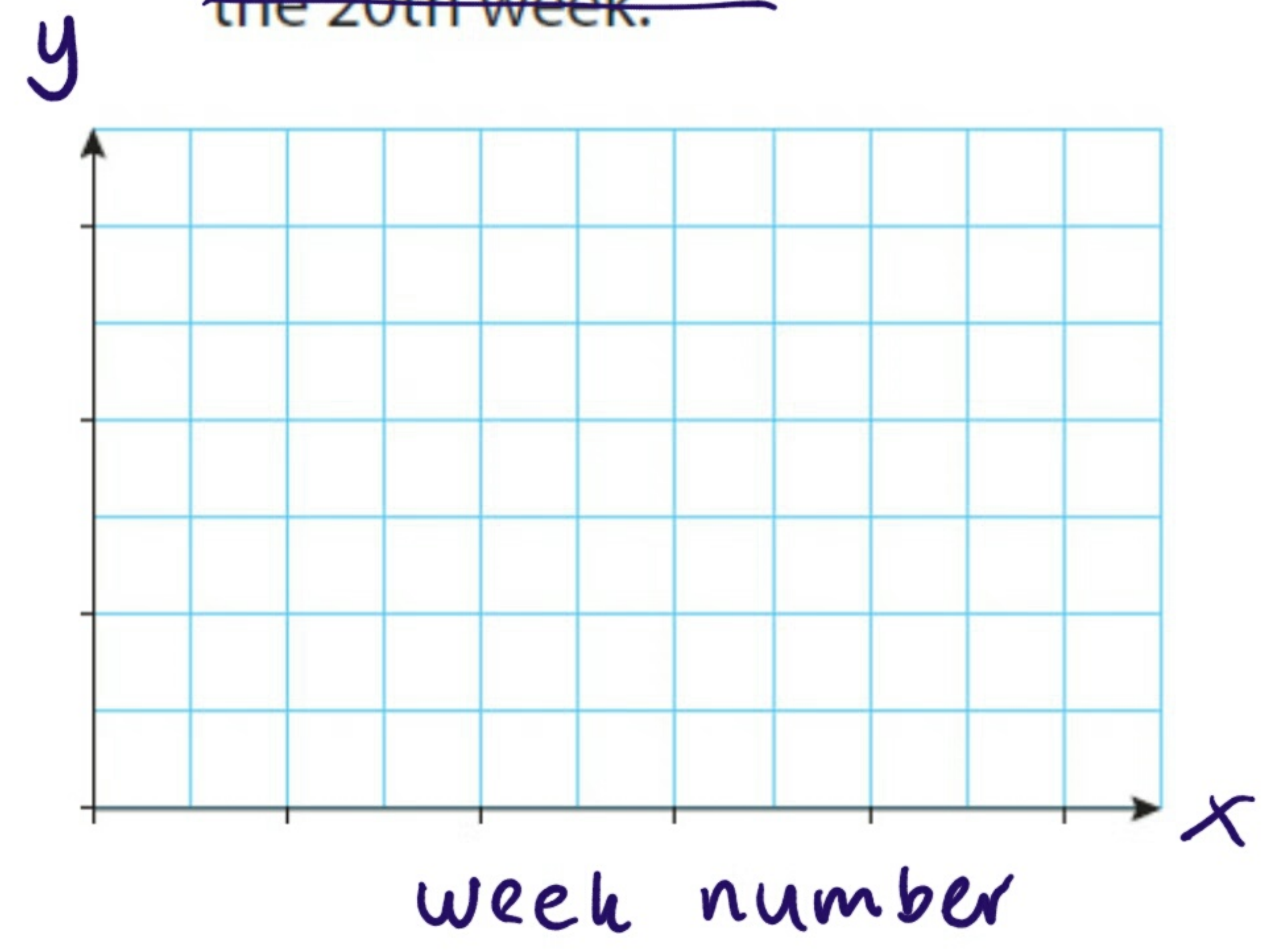
HIW
Q4 → 9
Pg 376 → 377

5. This pattern is made from cubes.

The outside faces of the cubes in each block are painted.

The table below shows the number of faces painted in each block.

Block number (n)	1	2	3	4
Number of painted faces	6	10	14	18

(i) Explain why the sequence formed by the numbers of faces painted forms a linear sequence.

(ii) Now find an expression for the nth term of the sequence.

(iii) Draw a graph of the sequence, putting the block numbers on the horizontal axis.
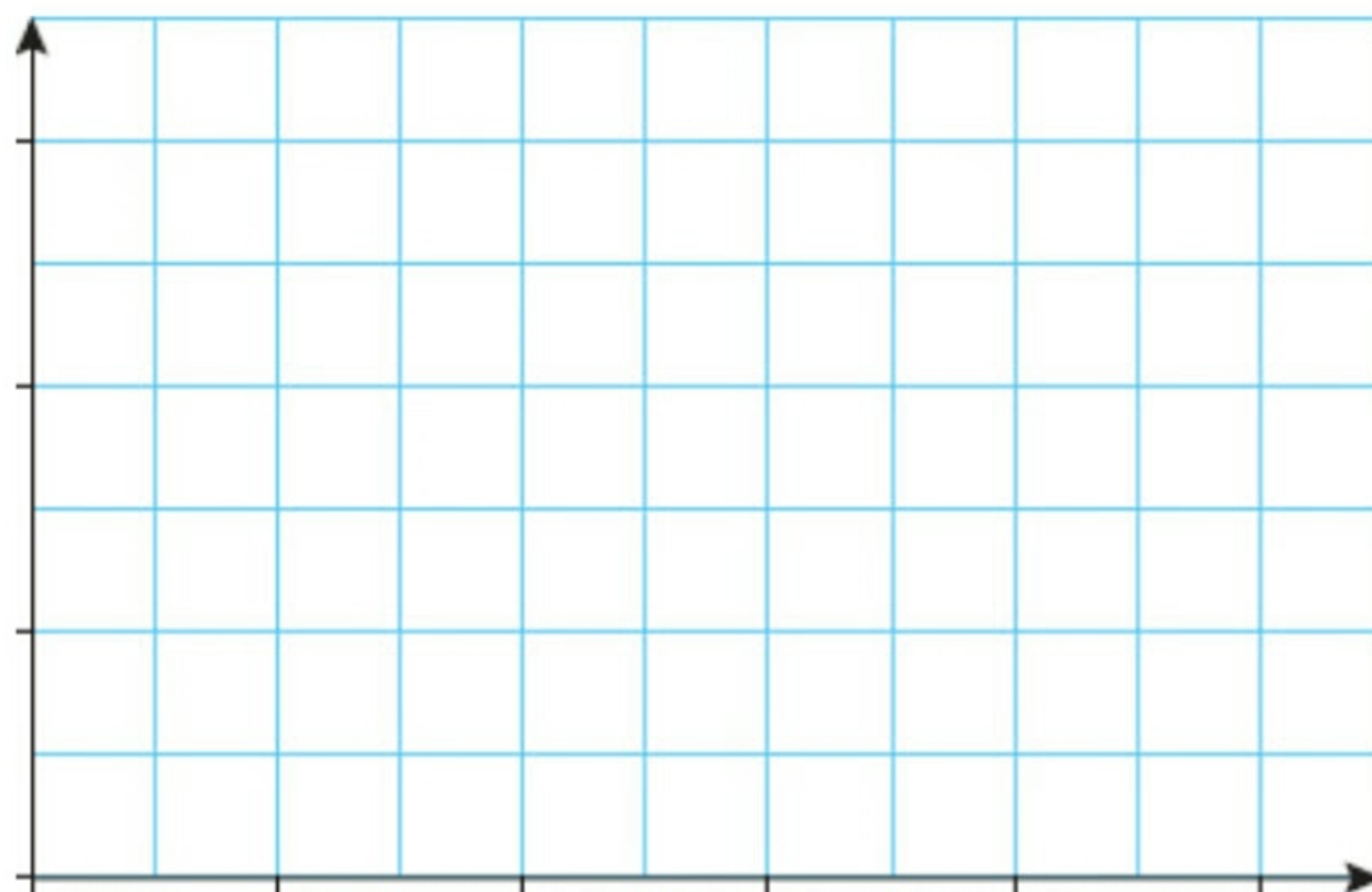
6. A fast-growing plant is 4 cm in height when purchased.
It grows 2 cm per day each day afterwords.
Copy and complete the table on the right showing
the height of the plant in its first seven days.

Day	Height (cm)
1	4
2	6
3	8
......	......

  (i) Draw a graph to show the height of the plant
      for Day 1 ..... Day 7.

 (ii) How many days will it take for the plant to
      reach a height of 30 cm?

(iii) The plant will stop growing when it reaches a height of 60 cm.
      How many days will this take?

(iv) What is the slope of the line you have drawn?

 (v) What is the rate of change of growth in the table?

(vi) What is the connection between your answers in (iv) and (v) above?
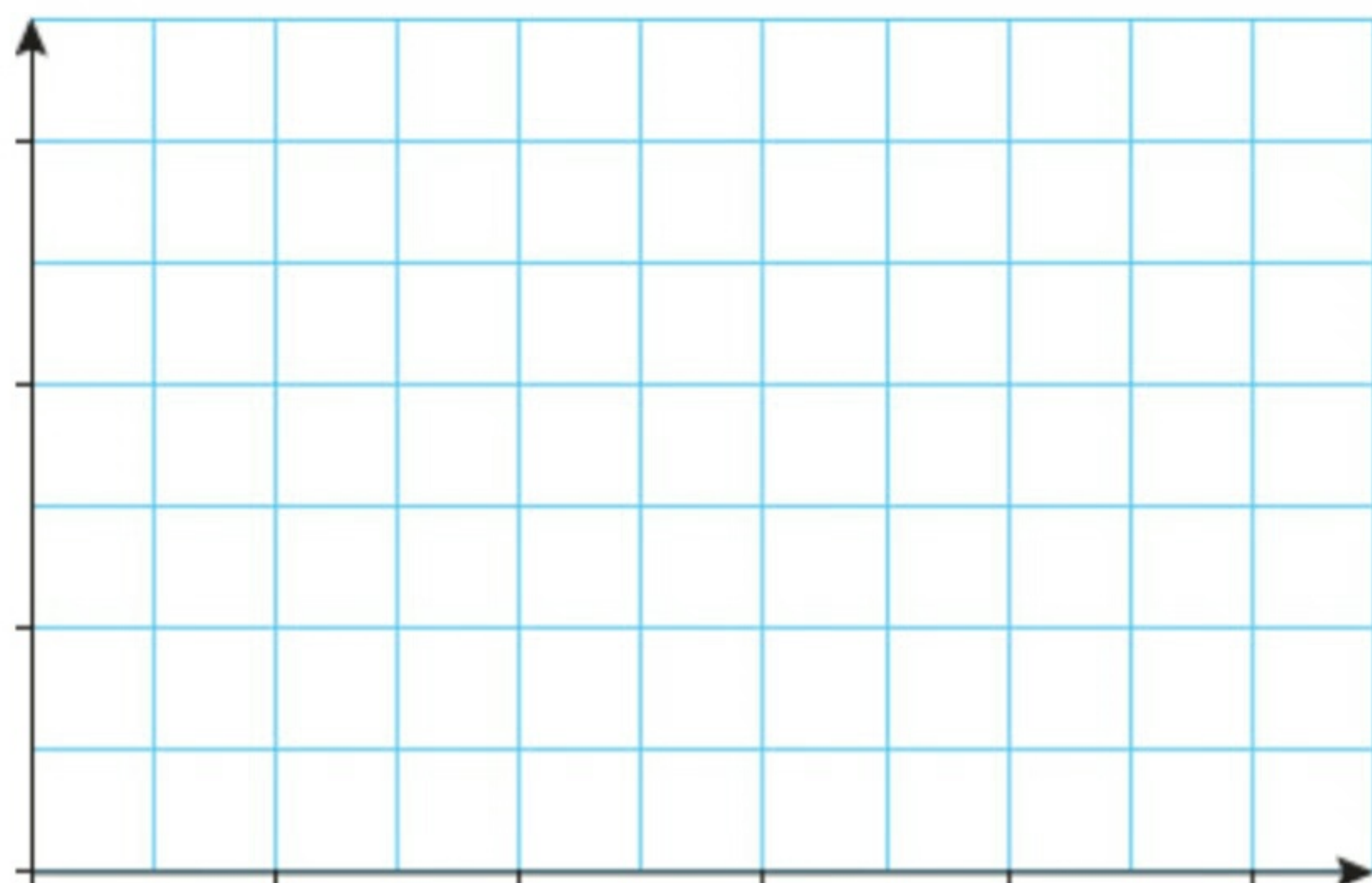
7. The numbers of edges for the numbers of hexagons are shown in the pattern and table below:
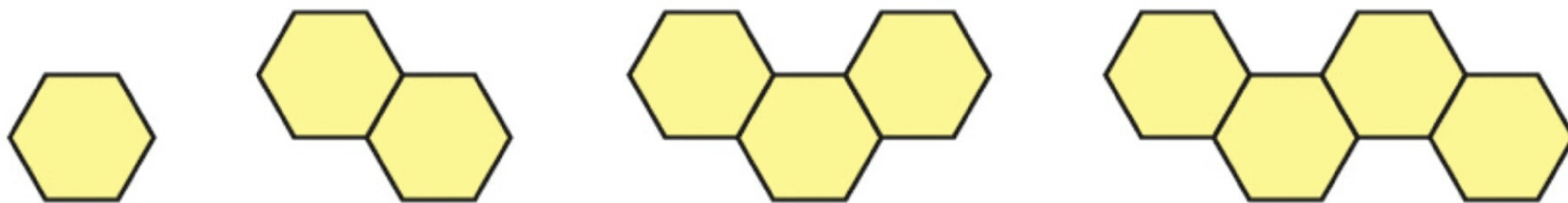
Number of hexagons (n)	1	2	3	4
Number of edges (E)	6	11	16	21

(i) Find the number of edges when there are 5 hexagons.

(ii) Explain why the pattern in the numbers of edges is linear.

(iii) Find an expression for the nth term of the sequence 6, 11, 16, …

7.

Number of hexagons (n)	1	2	3	4
Number of edges (E)	6	11	16	21

(iv) Draw a graph of the pattern, putting the numbers of hexagons on the x-axis.

 (v) Find the equation of the line you have drawn in the form $E = \boxed{}n + \boxed{}$.

(vi) What is the slope of this line?

(vii) Explain what the slope means in the context of the pattern.
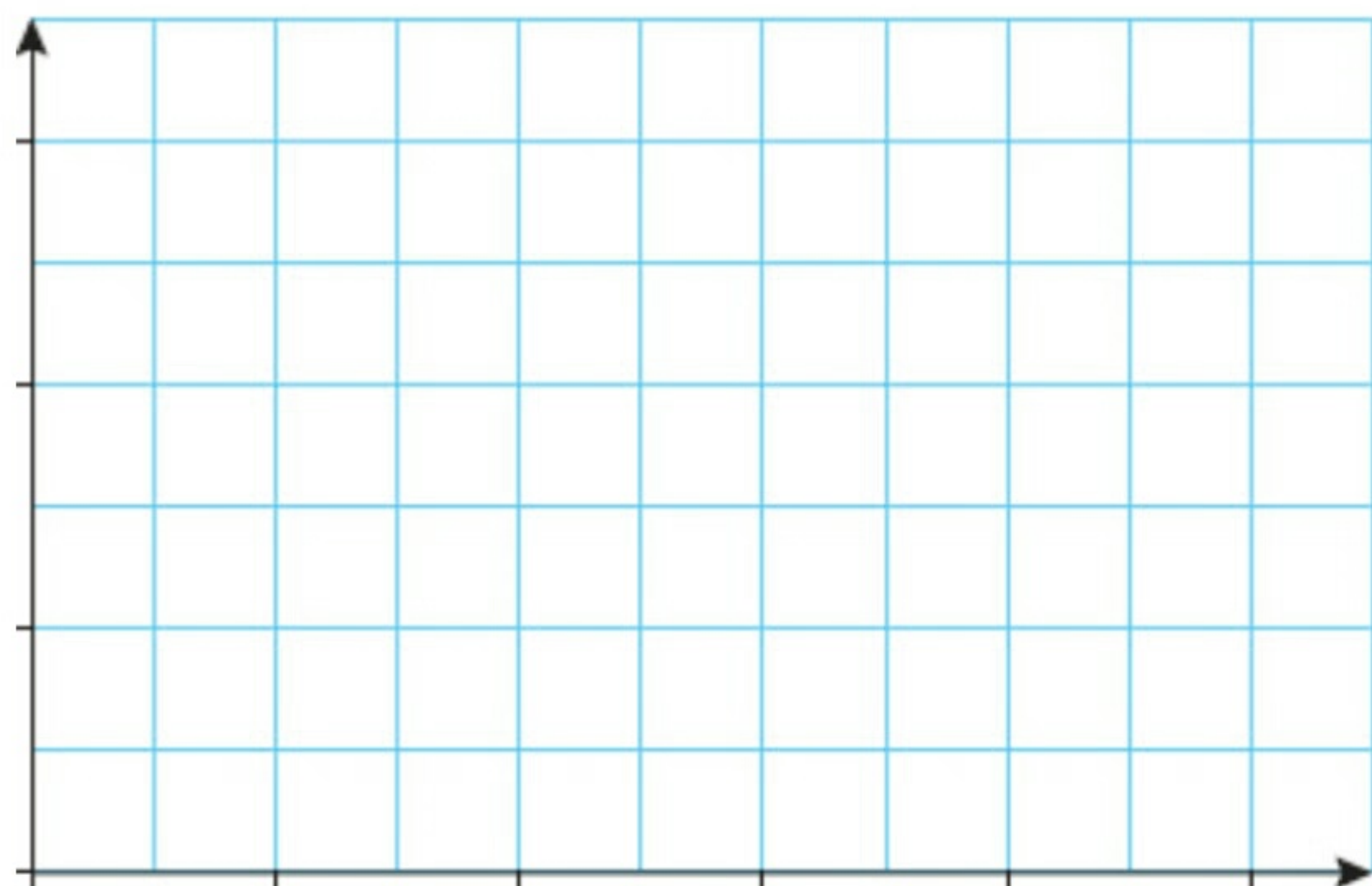
8. A sequence of numbers begins 4, 7, 12, 19, 28, …

   (i)   Explain why this is a quadratic pattern.

  (ii)   What is the second difference between the terms?

 (iii) Draw a graph of this pattern using
         the scales shown on the right.

 (iv) Describe the shape of the graph.
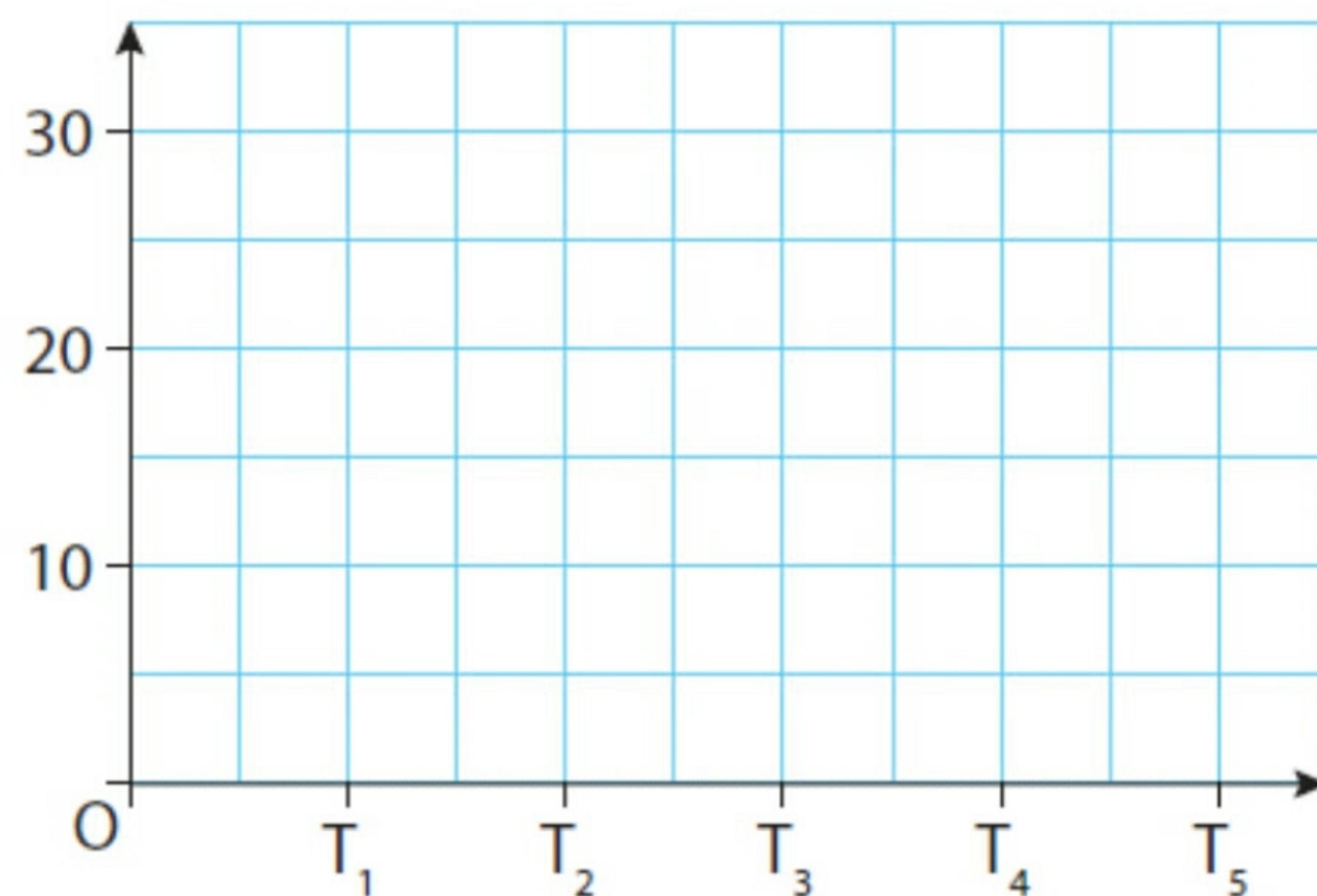
9. Here are the first three terms of an exponential sequence:

$$3, 9, 27, \ldots$$

(i) Write down the next three terms.

(ii) Investigate if the second difference is a constant.

(iii) Explain why the sequence is not quadratic.
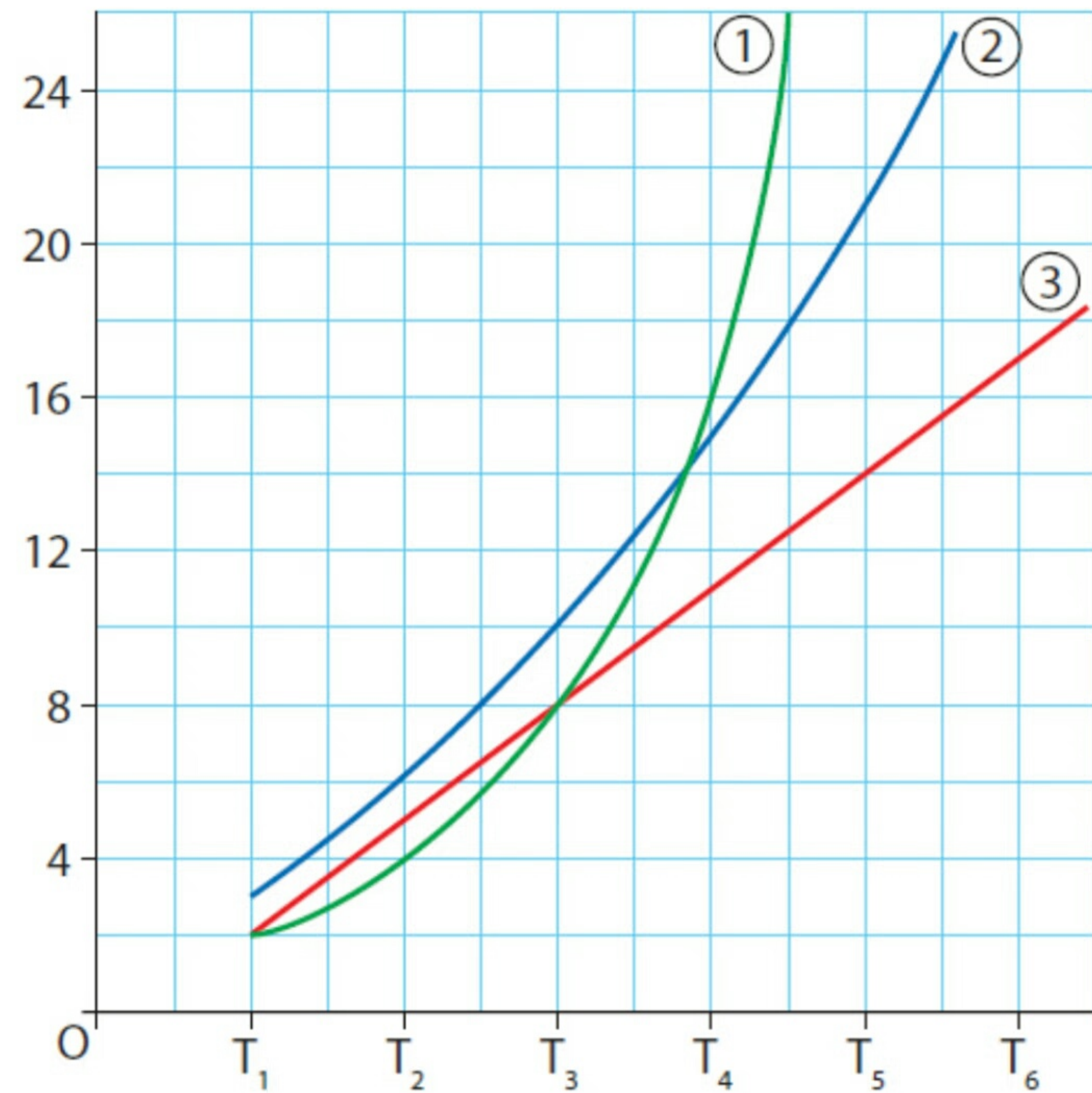
(iv) Describe one feature of an exponential graph.

10. Here are three graphs and three sequences.

Which sequence does each graph represent?

A: 2, 4, 8, 16, …

B: 3, 6, 10, 15, 21, …

C: 2, 5, 8, 11, …

Answers

Exercise 18.7

1. Constant difference between outputs
2. 3, 5, 7, 9, 11
3.   (i) €30   (ii) €105
   (iii) 5   (iv) Yes
    (v) €21   (vi) Yes, both 21
   (vii) €240
4.   (i) Yes   (ii) €120
   (iii) $y = 10x$ (or $a = 10w$)   (iv) €200
5.   (ii) $T_n = 4n + 2$
6.   (ii) 14   (iii) 29   (iv) 2
    (v) 2 cm per day
   (vi) The slope of the line equals the rate of growth of the plant

Answers

7. (i) 26 (iii) $T_n = 5n + 1$
   (v) $E = 5n + 1$ (vi) 5
   (vii) The increase in the number of edges from one pattern to the next

8. (ii) 2
   (iv) The graph is curved (parabola)

9. (i) 81, 243, 729
   (ii) It is not
   (iii) 2nd differences are not all the same
   (iv) The curve rises very steeply

10. A and 1, B and 2, C and 3

11. (i) $-4, -6, -6, -4, 0, 6$
    (iii) 5th term
    (iv) Curved (or parabola shaped)
    (v) 2nd and 3rd terms (or 1st and 4th)