chapter 18

Patterns and Sequences

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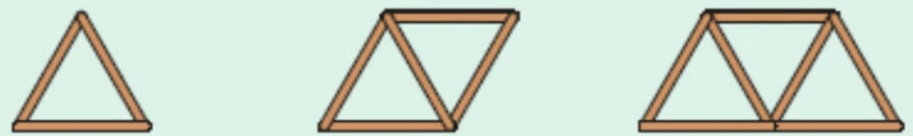
Section 18.5 Sequences formed from shapes

Example 1

The figure on the right shows some photo frames made with rods.



1 photo 2 photos 3 photos

(i) Draw the frame that holds 4 photos.

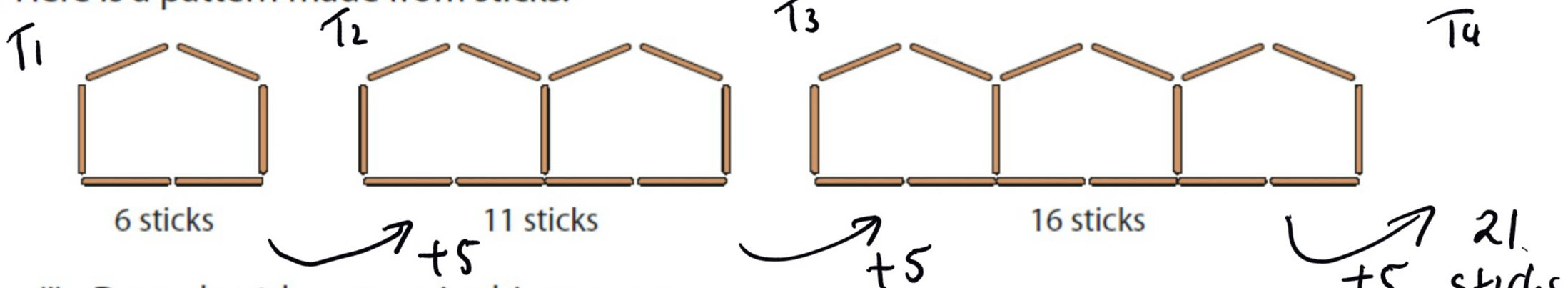
(ii) How many rods are there in the frame that holds 5 photos?

(iii) Find an expression for the number of rods in the *n*th frame.

(iv) Which frame uses 41 rods?

(v) Is it possible to make one of these frames using exactly 56 rods?

1. Here is a pattern made from sticks.



(i) Draw the 4th pattern in this sequence.

(ii) Write down the sequence of numbers generated by the sticks in the first six patterns.

6, 11, 16, 21, 26, 31, 36

(iii) Show that the number of sticks in the *n*th pattern is given by $T_n = 5n + 1$.

$T_n = a + (n-1)d$

$a = 6$

$d = +5$

$6 + (n-1)5$

$6 + 5n - 5$

$T_n = 5n + 1$

(iv) How many sticks are required for the 20th pattern?

$T_{20} = 5(20) + 1$

$100 + 1$

$T_{20} = 101$ sticks

(v) For which pattern are 51 sticks required?

Find n

$5n + 1 = 51$

$5(10) + 1 = 51$

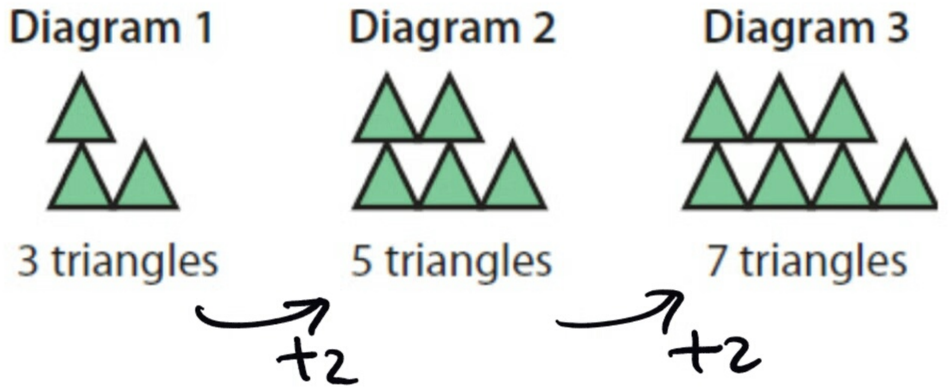
$T_{10} = 51$

2. Here are three diagrams made with triangles.

T_3 T_4 T_5 T_6 T_7

3, 5, 7, 9, 11, 13, 15

+2 +2 +2 +2



(i) Draw diagram 4.

(ii) How many triangles will be in diagram 7? 15

(iii) Find an expression for the number of triangles in the *n*th diagram.

(iv) Which diagram will contain 33 triangles?

iv) Δ

$2n+1=33$

$2(16)+1=33$

$32+1=33$

$T_{16}=33$

iii) Trial

$T_n = \boxed{+2}\,n \pm \square$

$T_1 = 2(1) \pm \square = 3$

$2+1=3$

$T_n = 2n+1$

$T_n = a+(n-1)d$

$a=3$

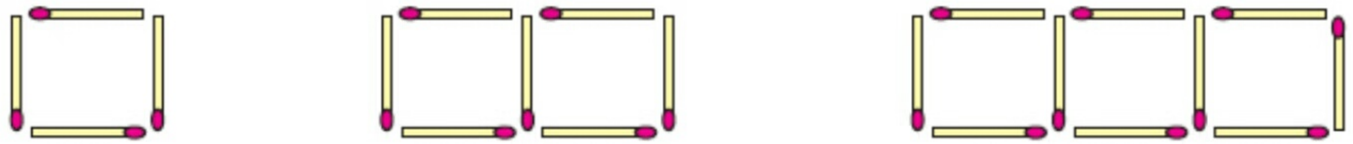
$d=2$

$3+(n-1)2$ ← multiply

$3+2n-2$

$T_n = 2n+1$

3. Complete the table of values for this sequence of matchstick patterns.



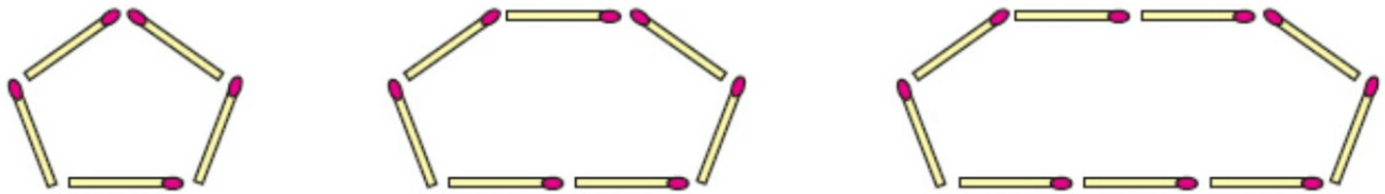
Number of squares	1	2	3	4	5
Number of matchsticks	4	7			

(i) How many matchsticks are required for the 6th pattern?

(ii) Find an expression in n for the number of matchsticks in the nth pattern.

(iii) Use the expression found to ascertain the number of matchsticks required for the 50th pattern.

4. Here is a pattern made with matchsticks.



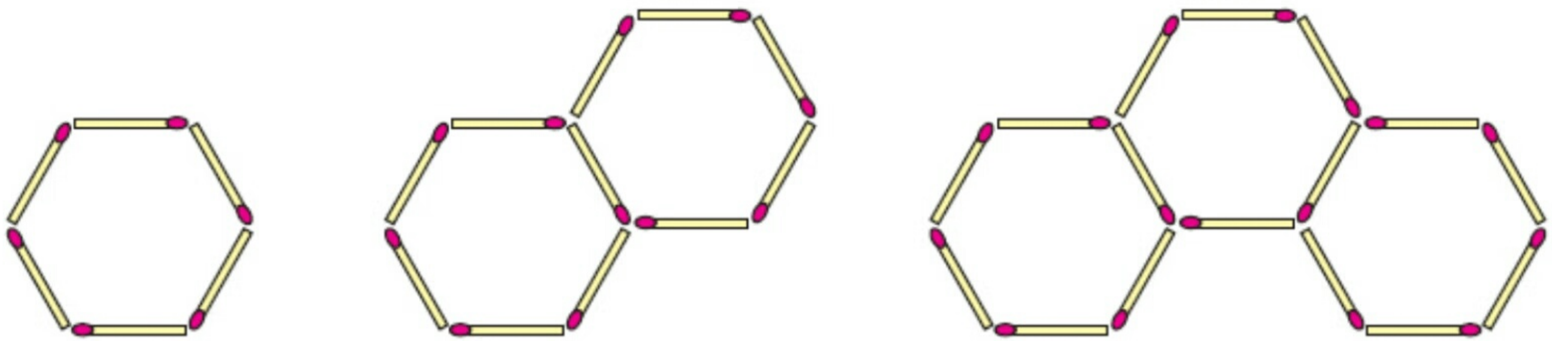
Pattern 1 Pattern 2 Pattern 3

(i) How many matchsticks will be in Pattern 5?

(ii) Find an expression for the number of matchsticks in the *n*th pattern.

(iii) In which pattern are there 51 matchsticks?

5. Here is another pattern made with matchsticks.



Pattern 1 Pattern 2 Pattern 3

(i) How many matchsticks will there be in pattern 4?

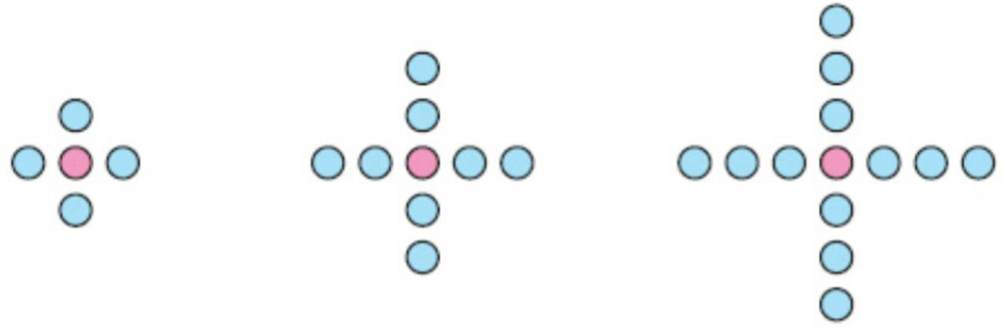
(ii) What is the term-to-term rule for the pattern?

(iii) How many matchsticks will there be in the *n*th pattern?

(iv) Which pattern will contain exactly 66 matchsticks?

(v) Will any pattern use exactly 88 matchsticks?

6. Sharon drew these shapes on her bedroom wall.



Shape 1 Shape 2 Shape 3

(i) Draw shape 4.

(ii) Copy and complete this table.

Shape number	1	2	3	4	5
Total number of circles	5	9			

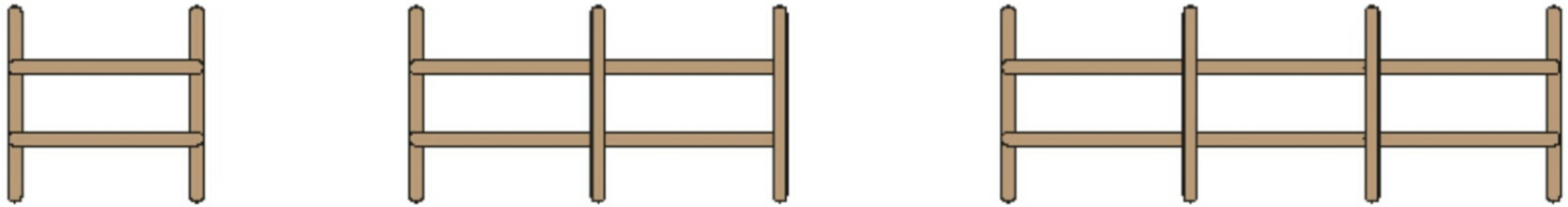
(iii) Describe in words the sequence formed.

(iv) Find an expression for the *n*th term of the sequence.

(v) Which shape will contain exactly 81 circles?

(vi) Will any shape contain exactly 89 circles?

7. Tommy builds fences of different lengths using pieces of wood.



Fence length 1 Fence length 2 Fence length 3

(i) Sketch fence length 5.

Tommy counted how many pieces he needed to make each fence length.
He then drew up the table below.

Fence length	1	2	3	4	5	6
Number of pieces	4	7	10			

(ii) Complete the table to show how many pieces of wood he would use for fence lengths 4, 5 and 6.

(iii) Write down, in terms of n, an expression for the number of pieces of wood needed for fence length n.

(iv) How many pieces of wood are needed for fence length 40?

(v) If 91 pieces of wood are needed, what is the number of the fence length?

Answers

Exercise 18.5

1. (ii) 6, 11, 16,, 21, 26, 31
(iv) 101 (v) 10th

2. (ii) 15 (iii) $2n + 1$ (iv) Diagram 16

3. (i) 19 (ii) $3n + 1$ (iii) 151

4. (i) 13 (ii) $2n + 3$ (iii) Pattern 24

5. (i) 21 (ii) Add 5 (iii) $5n + 1$
(iv) 13 (v) No

6. (iii) Start at 5 and add 4 (iv) $T_n = 4n + 1$
(v) Shape 20
(vi) Yes; Shape 22

7. (iii) $3n + 1$ (iv) 121 (v) 30