chapter 18

Patterns and Sequences

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Section 18.4 Finding the *n*th term of a sequence

Log tables pg 22

formula = $T_n = a + (n-1)d$

a = first term
d = common difference.

Example 1

Find the *n*th term of the sequence 3, 7, 11, 15, …

Example 2

Find the nth term of the sequence $10, 7, 4, 1, -2, \ldots$

Hence work out T_{20} of the sequence.

Example 3

The *n*th term of a sequence is given by $T_n = 5n - 4$.
Which term of the sequence is 21?

Exercise 18.4

1. A given sequence is 5, 9, 13, 17, …

+4 +4 +4

add 4.

(i) Write down the constant difference between the terms. +4

(ii) If $T_n = \boxed{4}\,n \pm$ number, what number goes in the box?

(iii) Now find an expression for T_n, the nth term.

(iv) Write down the value of T_{20}.

$T_1 = 4(1) \pm \square = 5$

$4 + 1 = 5$

$T_n = 4n + 1$

$T_2 = 4(2) \pm \square = 9$

$8 + 1 = 9$

$T_{20} = 4(20) + 1$

$80 + 1$

$T_{20} = 81$

Formula: $T_n = a + (n-1)d$

$a = 5,\ d = +4$

$T_n = 5 + (n-1)(4)$

$5 + 4n - 4$

$T_n = 4n + 1$

2. Find an expression for the *n*th term of each of these sequences: find T_n Pg 365

(i) 5, 7, 9, 11, …

a

+2 +2 +2

$d = 2$

$T_n = \boxed{2}n \pm \boxed{3}$

$T_1 = 2(1) \pm \square = 5$

$2 \pm \boxed{3} = 5$

$T_2 = 2(2) \pm \square = 7$

$4 \pm \boxed{3} = 3$

$T_n = 2n + 3$

$T_n = a + (n-1)d$

$T_n = 5 + (n-1)2$

$5 + 2n - 2$

H/W

(ii) 4, 7, 10, 13, …

+3 +3 3

$T_n = \boxed{3}n \pm \square$

$T_1 = 3(1) \pm \square = 4$

$3 \pm \boxed{1} = 4$

$T_n = 3n + 1$

$a = 4$

$d = 3$

$4 + (n-1)(3)$

$4 + 3n - 3$

$T_n = 3n + 1$

H/W

(iii) 6, 10, 14, 18, …

+4 +4 +4

$T_n = \boxed{4}n \pm \square = 6$

$T_1 = 4(1) \pm \square = 6$

$4 + 2 = 6$

$T_n = 4n + 2$

$a = 6$

$d = 4$

$a + (n-1)d$

$6 + (n-1)4$

$T_n = 4n + 2$

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3. Find an expression for the *n*th term of this sequence:

7, 11, 15, 19, ...

+4 +4 +4

Use the expression for the *n*th term to find T_{10} and T_{20}.

Find T_n

$T_n = \boxed{4}n \pm \square = 7$ (4 = d, some number)

$T_1 = 4(1) + \boxed{3} = 7$

$T_n = 4n + 3$

$T_{10} = 4(10) + 3$

$40 + 3$

$T_{10} = 43$

$T_{20} = 4(20) + 3$

$80 + 3 = 83$

$T_n = a + (n-1)d$

$a = 7$

$d = 4$

$7 + (n-1)4$

$7 + 4n - 4$

$T_n = 4n + 3$

4. Consider the sequence 12, 10, 8, 6, ...

constant diff $= -2$

(i) What is the term-to-term rule for this sequence?

(ii) If $T_n = -2n \pm$ a number, what number goes in the box?

$T_n = -2n \pm \square$

(iii) Use this to find an expression for T_n.

$T_n = -2n + 14$

(iv) Find T_{10} of the sequence.

(v) Which term ($n = ?$) of the sequence is -14?

$T_1 = -2(1) \pm \square = 12$

$-2 + 14 = 12$

$T_2 = -2(2) \pm \square = 10$

$-4 \pm 14 = 10$

$T_n = -2n + 14$

Formula $a = 12$, $d = -2$

$12 + (n-1)-2$

$12 - 2n + 2$

$T_n = -2n + 14$

$T_{10} = -2(10) + 14$

$-20 + 14$

$T_{10} = -6.$

v) $T_n \Rightarrow -2n + 14 = -14$ find n.

$-2n = -14 - 14$

$-2n = -28$

$n = 14.$

5. Find an expression for the *n*th term of these sequences:

(i) −3, 0, 3, 6, 9, …

+3 +3 +3 ✱

$T_n = a + (n-1)d$ ← multiply

$a = -3$

$d = +3$

$-3 + (n-1)3$

$-3 + 3n - 3 = T_n = 3n - 6$

Trial => $T_n = \boxed{3}n \pm \square$ ✱ T_n

$T_1 = 3(1) \pm \square = -3$

$3 - 6 = -3$

$-3 = -3$

$T_n = 3n - 6$

H/W (ii) 20, 15, 10, 5, …

3. Find an expression for the nth term of this sequence:

7, 11, 15, 19, ...

Use the expression for the nth term to find T_{10} and T_{20}.

4. Consider the sequence 12, 10, 8, 6, ...

(i) What is the term-to-term rule for this sequence?

(ii) If $T_n = \square n \pm$ a number, what number goes in the box?

(iii) Use this to find an expression for T_n.

(iv) Find T_{10} of the sequence.

(v) Which term of the sequence is -14?

5. Find an expression for the nth term of these sequences:

(i) $-3, 0, 3, 6, 9, \ldots$ (ii) $20, 15, 10, 5, \ldots$

6. Find an expression for T_n of the sequence 8, 5, 2,

For what value of n is $T_n = -34$?

Section 18.5 Sequences formed from shapes

So far in this chapter we have dealt only with number patterns.

In this section we will examine some geometric figures and the patterns they form.

Example 1

The figure on the right shows some photo frames made with rods.

(i) Draw the frame that holds 4 photos.

(ii) How many rods are there in the frame that holds 5 photos?

(iii) Find an expression for the number of rods in the nth frame.

(iv) Which frame uses 41 rods?

(v) Is it possible to make one of these frames using exactly 56 rods?

(i) This is the frame that holds 4 photos:

(ii) The sequence is 3, 5, 7, 9, 11, ...

The 5th frame has 11 rods.

(iii) The difference between the terms is +2.

Thus, the nth term will be $2n \pm$ a number.

If $T_n = 2n$, $T_1 = 2$ and so 1 must be added to get the first term 3.

$\therefore T_n = 2n + 1$

(iv) Let $T_n = 41 \Rightarrow 2n + 1 = 41$

$2n = 40$

$n = 20$

The 20th frame uses 41 rods.

(v) Let $T_n = 56$

$2n + 1 = 56$

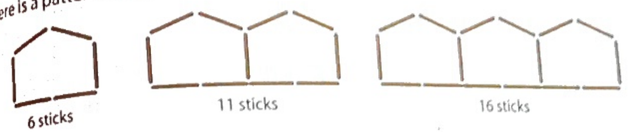
$2n = 55$

$n = 27\frac{1}{2}$

Since $27\frac{1}{2}$ is not a whole number, no frame uses exactly 56 rods.

Exercise 18.5

1. Here is a pattern made from sticks.



(i) Draw the 4th pattern in this sequence.

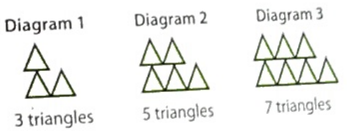
(ii) Write down the sequence of numbers generated by the sticks in the first six patterns.

(iii) Show that the number of sticks in the nth pattern is given by $T_n = 5n + 1$.

(iv) How many sticks are required for the 20th pattern?

(v) For which pattern are 51 sticks required?

2. Here are three diagrams made with triangles.



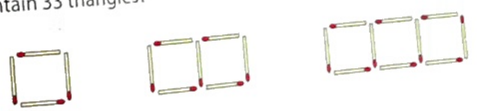
(i) Draw diagram 4.

(ii) How many triangles will be in diagram 7?

(iii) Find an expression for the number of triangles in the nth diagram.

(iv) Which diagram will contain 33 triangles?

3. Complete the table of values for this sequence of matchstick



H/W

6. Find an expression for T_n of the sequence 8, 5, 2, …
For what value of n is $T_n = -34$?

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Answers

Exercise 18.4

1. (i) $+4$ (ii) 4
(iii) $4n + 1$ (iv) 81

2. (i) $2n + 3$ (ii) $3n + 1$
(iii) $4n + 2$

3. $4n + 3$; 43, 83

4. (i) Subtract 2 (ii) -2
(iii) $-2n + 14$ (iv) -6
(v) T_{14}

5. (i) $3n - 6$ (ii) $-5n + 25$

6. $-3n + 11$; 15