

**Section 18.3 Linear sequences**

Term 1, Term 2, Term 3,  $T_4$ ,  $T_5$ , ...

common difference      will be      constant.

## Example 1

The  $n$ th term of a sequence is given by  $T_n = 3n - 4$ .

- (i) Write down the first three terms of the sequence and also  $T_{20}$ .
- (ii) Explain why the sequence is linear.

## Exercise 18.3

1. The  $n$ th <sup>rule</sup> terms of some sequences are given.

Write out the first four terms of each sequence.

$n$ th term any term in the pattern  $T_n$

(i)  $T_n = 2n$

$$\begin{aligned} T_1 &= 2(1) = 2 \\ T_2 &= 2(2) = 4 \\ T_3 &= 2(3) = 6 \\ T_4 &= 2(4) = 8 \\ T_{100} &= 2(100) = 200 \end{aligned}$$

2, 4, 6, 8  
↗ ↘  
+2 +2 constant common difference.

(ii)  $T_n = 3n + 1$        $n = 1, 2, 3, 4$  Terms

$$\begin{aligned} T_1 &= 3(1) + 1 = 4 \\ T_2 &= 3(2) + 1 = 7 \\ T_3 &= 3(3) + 1 = 10 \\ T_4 &= 3(4) + 1 = 13 \end{aligned}$$

Pattern

$$\begin{array}{cccc} T_1 & T_2 & T_3 & T_4 \\ 4 & 7 & 10 & 13 \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ +3 & +3 & +3 & \end{array}$$

(iii)  $T_n = 4n - 3$

$$\begin{aligned} T_1 &= 4(1) - 3 = 1 \\ T_2 &= 4(2) - 3 = 5 \\ T_3 &= 4(3) - 3 = 9 \\ T_4 &= 4(4) - 3 = 13 \end{aligned}$$

Pattern

$$\begin{array}{cccc} 1 & 5 & 9 & 13 \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ +4 & +4 & +4 & \end{array}$$

## Exercise 18.3

1. The  $n$ th terms of some sequences are given.  
Write out the first four terms of each sequence.

(iv)  $T_n = 2n + 5$

$$T_1 = 2(1) + 5 = 7$$

$$T_2 = 2(2) + 5 = 9$$

$$T_3 = 2(3) + 5 = 11$$

$$T_4 = 2(4) + 5 = 13$$

$$7, 9, 11, 13$$

↘ ↘ ↘  
+2 +2 +2

(v)  $T_n = 5n - 4$

$$T_1 = 5(1) - 4 = 1$$

$$T_2 = 5(2) - 4 = 6$$

$$T_3 = 5(3) - 4 = 11$$

$$T_4 = 5(4) - 4 = 16$$

$$1, 6, 11, 16$$

↘ ↘ ↘  
+5 +5 +5

(vi)  $T_n = 7 - 2n$

$$T_1 = 7 - 2(1) = 5$$

$$T_2 = 7 - 2(2) = 3$$

$$T_3 = 7 - 2(3) = 1$$

$$T_4 = 7 - 2(4)$$
$$7 - 8 = -1$$

$$5, 3, 1, -1$$

↘ ↘ ↘  
-2 -2 -2

B I R D M A S  
2<sup>2</sup> √  
3√

$$\sqrt{a} = a^{1/2}$$

$$\sqrt[3]{a} = a^{1/3}$$



3. The  $n$ th term of a sequence is  $T_n = 2n + 3$ . Rule.

(i) Write down the first five terms of the sequence.

(ii) Find  $T_{20}$  and  $T_{100}$ .

$$T_1 = 2(1) + 3 = 5$$

$$T_2 = 2(2) + 3 = 7$$

$$T_3 = 2(3) + 3 = 9$$

$$T_4 = 2(4) + 3 = 11$$

$$T_5 = 2(5) + 3 = 13$$

$$T_{20} = 2(20) + 3$$
$$40 + 3 = 43$$

$$T_{20} = 43$$

$$T_{100} = 2(100) + 3$$
$$200 + 3$$

$$= 203$$
$$T_{100} = 203$$

4. If  $T_n = 2n - 6$ , show that  $T_1 + T_5 = 0$ .

$$T_1 = 2(1) - 6 = -4$$

$$T_5 = 2(5) - 6 = 4$$

$$\begin{array}{r} T_1 + T_5 \\ -4 + 4 = 0 \end{array}$$



5. Explain why each of these sequences is linear:

(i) 3, 8, 13, 18, ...

$\curvearrowright \curvearrowright \curvearrowright$   
+5 +5 +5

first diff  
is  
constant

(ii) 18, 14, 10, 6, ...

$\curvearrowright \curvearrowright \curvearrowright$   
-4 -4 -4

first diff  
is  
constant

H/W

6. Investigate whether each of these sequences is linear:

(i)  $-8, -10, -12, -14, \dots$

(ii)  $1, 3, 9, 27, \dots$

(iii)  $20, 10, 5, 2\frac{1}{2}, \dots$

(iv)  $0.1, 0.3, 0.5, 0.7, \dots$

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7. The  $n$ th terms of six different sequences are:

**A**

$$7n - 2$$

**B**

$$10 - n$$

**C**

$$\frac{1}{2}n - 3$$

**D**

$$n^2 + 1$$

**E**

$$\frac{60}{n}$$

**F**

$$2^n$$

- (i) Calculate the first four terms of each sequence.
- (ii) Calculate the 20th term of each sequence.
- (iii) Which of these sequences are linear?

HW

8. The  $n$ th term of a sequence is  $3n + 2$ .

- (i) Write down the first six terms of the sequence.
- (ii) Calculate the 100th term.

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9. Linear sequences can be found on this grid. Two are shown on the diagram.

- (i) Find seven more linear sequences that have four terms or more.

Write down each sequence as an **increasing** sequence and find its next term.

44	34	24	14	4	3	6	9	12
40	30	5	20	10	11	5	8	1
44	37	30	23	16	9	2	7	3
4	11	23	21	22	12	1	6	9
1	7	26	20	28	9	8	5	0
3	31	10	15	34	30	12	4	8
36	6	11	13	40	0	1	3	2

- (ii) The expressions below give the  $n$ th terms of these sequences.

Match each expression to its sequence.

$$3n$$

$$6n - 2$$

$$10n - 6$$

$$2n + 1$$

$$4n$$

$$3n + 1$$

$$n + 2$$

$$5n + 1$$

$$7n - 5$$

# Answers

## Exercise 18.3

1. (i) 2, 4, 6, 8 (ii) 4, 7, 10, 13  
(iii) 1, 5, 9, 13 (iv) 7, 9, 11, 13  
(v) 1, 6, 11, 16 (vi) 5, 3, 1, -1
2. (i) -2, -5, -8 (ii) 2, 5, 10  
(iii)  $\frac{1}{2}$ ,  $\frac{3}{4}$ , 1
3. (i) 5, 7, 9, 11, 13 (ii) 43, 203
5. As the difference between the terms is a constant
6. (i) Yes (ii) No  
(iii) No (iv) Yes

# Answers

- 7.** (i) A – 5, 12, 19, 26  
B – 9, 8, 7, 6  
C –  $-2\frac{1}{2}$ ,  $-2$ ,  $-1\frac{1}{2}$ ,  $-1$   
D – 2, 5, 10, 17  
E – 60, 30, 20, 15  
F – 2, 4, 8, 16
- (ii) A – 138  
B –  $-10$   
C – 7  
D – 401  
E – 3  
F – 1048576
- (iii) A, B, C are linear

**8.** (i) 5, 8, 11, 14, 17, 20

(ii) 302