18 Patterns and Sequences

Section 18.3 Linear sequences

Term 1, Term 2, Term 3, T4, T5,...

common Will constant.

difference be

361

Example 1

The *n*th term of a sequence is given by $T_n = 3n - 4$.

- (i) Write down the first three terms of the sequence and also T_{20} .
- (ii) Explain why the sequence is linear.

Exercise 18.3

1. The <u>nth terms</u> of some sequences are given.

Write out the first four terms of each sequence.

nth term any term in the pattern In

(i)
$$T_n = 2n$$
 $T_1 = 2(1) = 2$ $2, 4, 6, 8$ $T_2 = 2(2) = 4$ $+2+2$ constand common difference. $T_3 = 2(3) = 6$ $t = 8(4) = 8$ $t = 8(4) = 8$

(ii)
$$T_n = 3n + 1$$
 $n = 1, 2, 3, 4$ Terms
$$T_1 = 3(1) + 1 = 4$$

$$T_2 = 3(2) + 1 = 7$$

$$T_3 = 3(3) + 1 = 10$$

$$T_4 = 3(4) + 1 = 13$$

$$T_5 = 3(3) + 1 = 10$$

$$T_7 = 3(4) + 1 = 13$$

(iii)
$$T_n = 4n - 3$$

 $T_1 = 4(1) - 3 = 1$
 $T_2 = 4(2) - 3 = 5$
 $T_3 = 4(3) - 3 = 9$
 $T_4 = 4(4) - 3 = 13$

Exercise 18.3

1. The *n*th terms of some sequences are given. Write out the first four terms of each sequence.

(iv)
$$T_n = 2n + 5$$

 $T_1 = 2(1) + 5 = 7$
 $T_2 = 2(2) + 5 = 9$
 $T_3 = 2(3) + 5 = 11$
 $T_4 = 2(4) + 5 = 13$

(v)
$$T_n = 5n - 4$$

 $T_1 = 5(1) - 4 = 1$
 $T_2 = 5(2) - 4 = 6$
 $T_3 = 5(3) - 4 = 11$
 $T_4 = 5(4) - 4 = 16$

(vi)
$$T_n = 7 - 2n$$

 $T_n = 7 - 2n$
 $T_n = 7 - 2(1) = 5$
 $T_n = 7 - 2(1) = 5$

2. Write out the first three terms of these sequences defined by the given nth term:

(i)
$$T_n = 1 - 3n$$
 $T_1 = 1 - 3(1) = -2$

$$T_2 = 1 - 3(2) = -2 - 3 = -5$$

$$1 - 6 = -5$$

$$T_3 = 1 - 3(3)$$

$$1 - 9 = -8$$

(iii)
$$T_n = n^2 + 1$$
 $T_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}^2 + 1$ $T_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}^2 + 1$ $T_3 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}^2 + 1$ $T_4 = 10$ $T_5 = 10$ $T_6 = 10$ $T_7 = 10$

(iii)
$$T_n = \frac{n+1}{4}$$
 $T_1 = \frac{(1)+1}{4} = \frac{2}{4} = \frac{1}{2}$
 $T_2 = \frac{(2)+1}{4} = \frac{3}{4}$
 $T_3 = \frac{(3)+1}{4} = \frac{4}{4} = 1$

3. The *n*th term of a sequence is $T_n = 2n + 3$. Rule.

- (i) Write down the first five terms of the sequence.
- (ii) Find T_{20} and T_{100} .

$$T_1 = 2(1) + 3 = 5$$
 $T_{20} = 2(20) + 3$
 $T_{20} = 43$
 $T_{20} = 43$
 $T_{3} = 2(3) + 3 = 9$
 $T_{4} = 2(4) + 3 = 11$
 $T_{5} = 2(5) + 3 = 13$
 $T_{100} = 203$

4. If $T_n = 2n - 6$, show that $T_1 + T_5 = 0$.

$$I_1 = 2(1) - 6 = -4$$

$$T_5 = 2(5) - 6 = 4$$

$$T_1 + T_5$$

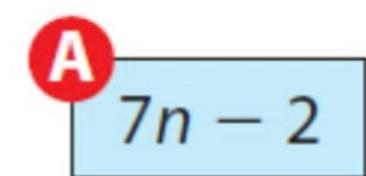
$$-4 + 4 = 0$$

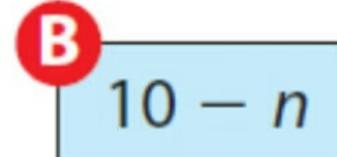
5. Explain why each of these sequences is linear:

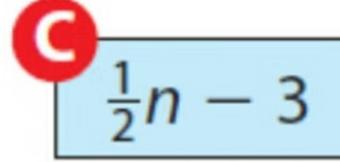
(i)
$$-8$$
, -10 , -12 , -14 , ...
(iii) 20 , 10 , 5 , $2\frac{1}{2}$, ...

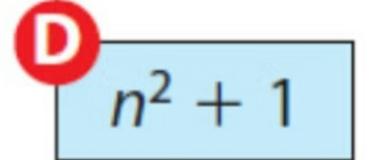
(iii) 20, 10, 5,
$$2\frac{1}{2}$$
, ...

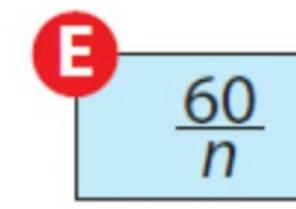
7. The nth terms of six different sequences are:













- (i) Calculate the first four terms of each sequence.
- (ii) Calculate the 20th term of each sequence.
- (iii) Which of these sequences are linear?

- (i) Write down the first six terms of the sequence.
- (ii) Calculate the 100th term.

- Linear sequences can be found on this grid.
 Two are shown on the diagram.
 - (i) Find seven more linear sequences that have four terms or more.

Write down each sequence as an **increasing** sequence and find its next term.

44	34	24	14	4	3/	6	9	12
40	30	5	20	10	11	15	8	1
44	37	30	23/	16	19	2	7	3
4	11	23/	21	122	12	1	6	9
1	7/	26	20	28	9	8	5	0
3/		10				12		8
(36)	6	11	13	40	0	1	3	2

(ii) The expressions below give the nth terms of these sequences.

Match each expression to its sequence.

3*n*

6n - 2

10n - 6

2n + 1

4*n*

3n + 1

n+2

5n + 1

7n - 5

Answers

Exercise 18.3

- **1.** (i) 2, 4, 6, 8
 - (iii) 1, 5, 9, 13
 - (v) 1, 6, 11, 16
- 2. (i) -2, -5, -8
 - (iii) $\frac{1}{2}$, $\frac{3}{4}$, 1
- **3.** (i) 5, 7, 9, 11, 13
- (ii) 43, 203

(ii) 4, 7, 10, 13

(iv) 7, 9, 11, 13

(vi) 5, 3, 1, -1

(ii) 2, 5, 10

- As the difference between the terms is a constant
- **6.** (i) Yes
 - (iii) No

- (ii) No
- (iv) Yes

Answers

7. (i)
$$A - 5$$
, 12, 19, 26
 $B - 9$, 8, 7, 6
 $C - -2\frac{1}{2}$, -2 , $-1\frac{1}{2}$, -1
 $D - 2$, 5, 10, 17
 $E - 60$, 30, 20, 15
 $F - 2$, 4, 8, 16
(ii) $A - 138$
 $B - -10$
 $C - 7$
 $D - 401$
 $E - 3$
 $F - 1048576$
(iii) A, B, C are linear

8. (i) 5, 8, 11, 14, 17, 20

(ii) 302