Sum of the series

Pg 293 Q2

$3+7+11+15+\dots$

$\underbrace{}_{+4}\ \underbrace{}_{+4}$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$a=3$

$d=4$

$$S_n = \frac{n}{2}[2(3)+(n-1)4]$$

$$\frac{n}{2}[6+4n-4]$$

$$\frac{n}{2}(4n+2)$$

$$S_{20} = \frac{20}{2}(4(20)+2)$$

$$\boxed{\begin{array}{l}\text{Pg } 293 \\ Q3 \rightarrow 10\end{array}}$$

$10(80+2)$

$10(82)$

$=820$

Q3) S_n

$a = 1$

$1 + 4 + 7 + 10 +$

$d = 3$

$+3$

$S_n = \frac{n}{2}\left(2(1) + (n-1)3\right)$

$= \frac{n}{2}\left(2 + 3n - 3\right)$

$S_n = \frac{n}{2}\left(3n - 1\right) \Rightarrow \frac{3n^2 - n}{2}$

$S_{16} = \frac{16}{2}\left(3(16) - 1\right) = 376$

Q4) $7 + 10 + 13 + 16 + \dots -$

$a = 7$

$d = 3$

$S_8 = \frac{\overset{n}{8}}{2}\left(2(\overset{a}{7}) + (\overset{n}{8} - 1)\overset{a}{3}\right)$

$S_8 = 140$

Q5) $16 + 12 + 8 + 4$

$a = 16$

$d = -4$

$S_{24} = \frac{24}{2}\left(2(16) + (24 - 1) - 4)\right)$

$S_{24} = -720$

Q6) $T_n = 5n - 2$

$a = 3$

$d = 5$

$T_1 = 5(1) - 2$

$T_1 = 3$

$T_2 = 5(2) - 2$

$10 - 2$

$T_2 = 8$

$\underbrace{3, 8}_{+5}$

$S_{16} = \frac{16}{2}\left(\underbrace{2(3) + (16 - 1) \times 5}\right)$

$S_{16} = 648$

7) $1 + 2 + 3 +$ $\frac{n}{2}(n+1)$

$a = 1$
$d = 1$

$S_n = \frac{n}{2}(2(1) + (n-1)1)$

$= \frac{n}{2}(2 + n - 1)$

$\frac{n}{2}(n+1)$

$S_{100} = \frac{100}{2}(100+1)$

$S_{100} = 5050$

Quadratic sequences

A quadratic pattern will have a first and a second difference. If the second difference is constant the pattern is quadratic

Eg) Find the next two terms of the quadratic sequence

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5$

3, 4, 6, 9, 13, ... 18 24

first diff: +1 +2 +3 +4 +5 +6

not constant
not linear

Second diff: +1 +1 +1

second diff is constant.
∴ quadratic

Eg2) Find the first five terms of the nth term

$T_n = n^2 + 4$

		first diff	second diff
First term :	$T_1 = (1)^2 + 4 = 5$		
		3	
2nd term	$T_2 = (2)^2 + 4 = 8$		2
		5	
	$T_3 = (3)^2 + 4 = 13$		2
		7	
	$T_4 = (4)^2 + 4 = 20$		2
		9	
	$T_5 = (5)^2 + 4 = 29$		

Linear pattern

$T_n = an + b$

$T_n = a + (n-1)d$

Quadratic

$T_n = an^2 + bn + c$

where a, b and $\in \mathbb{Z}$

Eg1 Find an expression for the n^{th} term of the sequence.

T_1, T_2

7, 10, 15, 22, 31 ....

3 5 7 9

2 2 2 second diff

Note: $a = \frac{1}{2}$ the second difference

$a = 2 / \tfrac{1}{2} = 1$

$a = 1$

$T_n = an^2 + bn + c$

$T_1 \Rightarrow (1)^2 + b(1) + c = 7$

make equation

$1 + b + c = 7$

$-1 \mid b + c = 6 \mid -1$

$T_2 \Rightarrow (2)^2 + b(2) + c = 10$

$4 + 2b + c = 10$

$-4 \mid 2b + c = 6 \mid -4$

Simultaneous Equation

(i) $b + c = 6$ (-1)

$2b + c = 6$

$\Rightarrow$

$-b - \cancel{c} = -6$

$2b + \cancel{c} = 6$

$b = 0$

Find c

$b + c = 6$

$b = 0$

$(0) + c = 6$

$c = 6$

$a = 1$

$b = 0$

$c = 6$

$T_n = an^2 + bn + c$

$T_n = n^2 + 6$

Pg 297

Q6