If all outcomes are equally likely to occur then the trial/experiment is said to be fair.

Eg. If you tossed a coin 100 time you would expect to get a Head 50% of the time and also a tail 50%

To determine in a trial is fair, it must be repeated a sufficiently large number of times.

Expected Frequency.

If the relative frequency/probability of the event is known we can estimate how many times the event would happen over a certain number of trials.

Formula Expected Frequency = $\left(\begin{array}{c}\text{number}\\\text{of trials}\end{array}\right) \times \left(\begin{array}{c}\text{Probability}\\\text{of the event}\end{array}\right)$

Eg 1) A six sided die is rolled 600 times the 1 is shown 150 times
a) Do you think it's a fair die? Explain your answer

P(1 on die) = 1/6 $\frac{150}{600} = \frac{1}{4}$

≈ 16%   <   ≈ 25%   Not a fair

HW
b) If the die is rolled another 200 times how many times in total would you expect to get a 1 on the die.

Pg 191 Q 1+2

Total = 600 + 200 = 800

(800) x (1/4) = 200

T&T2 10.5 Estimating... (PowerPoint icon)

T&T2 10.5 Estimating... (pdf icon)

Relative Frequency (experimental probability)

We work out the probability of an event occurring using the data taken from the experiment.

Relative frequency is an estimate of the probability of an event

Formula = $\dfrac{\text{Frequency (no. of times the event happened)}}{\text{Total number of trials.}}$

Eg 1) In a survey of 1000 households it was found that 650 households had a pet. If a household is selected at random, find the relative frequency that the household has a pet

$\frac{650}{1000} = \frac{13}{20}$ as % = 65%

fraction

Eg 2) Claire took 20 penalties last season. She scored 16 of the penalties. Find the relative frequency of her successful shots.

$\frac{16}{20} = \frac{4}{5}$ or   80%

Eg 3) Philip tosses a coin 10, 20, 30 and 200 times and records the results.

No. of tosses	10	20	30	200
no. of Heads	3	14	12	102
Relative frequency.	$\frac{3}{10}$	$\frac{14}{20} = .7$	$\frac{12}{30} =$	$\frac{102}{200}$

Probability **10**

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Example 1

Derek collects data on the colours of cars passing the school gate.
His results are shown in the table.

Colour	Tally	Frequency
White	JHT JHT JHT JHT IIII	24
Red	JHT JHT JHT JHT JHT JHT II	32
Black	JHT JHT IIII	14
Blue	JHT JHT JHT I	16
Green	JHT JHT	10
Other	IIII	4

(i)   How many cars did Derek survey?
(ii) What was the relative frequency of blue cars?
(iii) What was the relative frequency of red cars?
      Give your answer as a decimal.
(iv) Write down an estimate of the probability that the next car passing the school gate will be green.
(v)   How can the estimate for the probability of green cars be made more reliable?

Example 2

This spinner is biased.
The probability that the spinner will land on each of the numbers 1 to 4 is given in the table below.

Number	1	2	3	4	5
Probability	0.35	0.1	0.25	0.15	

The spinner is spun once.
(i)   Work out the probability that the spinner will land on 5.
(ii) Write down the number on which the spinner is most likely to land.
(iii) If the spinner is spun 200 times, how many times would you expect it to land on 3?

1. A fair coin is tossed 100 times.
   How many heads would you expect to get?

   $$100 \times \frac{1}{2} = 50$$

2. A fair six-sided dice is thrown 60 times.
   (i)   How many sixes would you expect to get? *10*
   (ii) How many twos would you expect to get? *10*
   (iii) How many even numbers would you expect to get? *30*

3. One ball is selected at random from the bag
   shown and then replaced. This procedure
   is done 400 times.
   How many times would you expect to select:
   (i)   a blue ball,
   (ii) a red ball?

4. 50 cars are observed passing the school gate. 15 of these cars are red.
   Use these results to estimate the probability that the next car to pass the school gate
   will be red.

5. Jane and Nicky both did the 'dropping a drawing pin' experiment.
   Here are their results.

   Jane
Trials	20
'Point up'	10

   Nicky
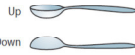 | Trials | 100 |
   |---|---|
   | 'Point up' | 75 |

   Another drawing pin is dropped.
   (i)   For Jane, what is the probability of getting 'point up'?
   (ii) For Nicky, what is the probability of getting 'point up'?
   (iii) Whose result is likely to be more reliable? Explain your answer.

6. Ciara surveyed the colours of vehicles passing her home. Here are her results.

Colour	Red	Black	Silver	Green	White	Other
Frequency	6	10	24	4	12	8

Calculate an estimate of the probability that the next vehicle to pass will be
(i) silver
(ii) not black.

7. Eoin is carrying out an experiment – 'dropping a spoon'.
He records whether the spoon lands up or down. Up
(i) Copy and complete this table of Eoin's results. Down

Number of drops	5	10	20	25	50	100
Number of times landed up	3	7	11	15	32	63
Probability	0.6	0.7				

(ii) What do you think could be a reasonable estimate for the true probability of a spoon landing up? Give your answer as a decimal, correct to 1 decimal place.
(iii) If Eoin dropped the spoon 300 times, how many times is it likely to land up? (i.e., the expected frequency.)

8. A bag contained coloured beads. Darren randomly selects a bead and then replaces it.
He does this 60 times. Here are the results.

Colour	White	Green	Blue
Frequency	10	30	20

Estimate the probability that on his next draw he will select:
(i) a white bead
(ii) a green bead
(iii) a white bead or a blue bead

9. A bag contains eight counters. Each counter is blue or red or yellow.
Helena takes a counter at random from the bag. She records the colour of the counter and then puts the counter back in the bag.
She performs this experiment 400 times. The table shows her results.

Colour of counter	Blue	Red	Yellow
Frequency	210	90	92

(i) Estimate how many of the eight counters are blue.
Give an explanation for your answer.
(ii) Estimate how many of the eight counters are yellow.
Give an explanation for your answer.

10. Olivia, Ben and Joe each rolled a different dice 360 times.

Only one of the dice was fair.
Whose was it?
Explain your answer.
Whose dice is the most biased?
Explain your answer.

Number	Olivia	Ben	Joe
1	27	58	141
2	69	62	52
3	78	63	56
4	43	57	53
5	76	56	53
6	67	64	5

11. A fair six-sided dice has its faces painted red, white or blue.
The dice is thrown 36 times.
Here are the results:

Colour	Frequency
Red	7
White	11
Blue	18

Based on the results, how many faces do you think are painted each colour?

12. Lyra made a spinner with three colours – yellow, blue and red.
She tested it by spinning it 500 times.

Her results were: yellow 240
blue 160
red 100

(i) Estimate the probability of the spinner landing on yellow.
(ii) She then spun the spinner 100 times.
About how many times would you expect the spinner to land on yellow?

13. Carol spins a red and a green spinner 50 times each. She records whether or not each spinner lands on a 1. The tables show her results.

Red spinner

Lands on a 1	Does not land on a 1
7	43

Green spinner

Lands on a 1	Does not land on a 1
24	26

(i) Work out the relative frequency of getting a 1 on each spinner.
(ii) Both spinners are fair six-sectored spinners.

How many sides of each spinner do you think will have a 1 on them?
Give an explanation for your answers.

14. The probability of having to wait for more than 5 minutes at a post office is $\frac{2}{7}$.
350 people use the post office in one day.
Work out an estimate for the number of people who have to wait for more than 5 minutes.

15. These are the results of an experiment to test a spinner:

(i) Explain what the spinner could look like.
(ii) Draw a possible sketch of the spinner.

	Result of spins
Red	50
White	30
Blue	20
Total	100

16. David and Rory are keen golfers

Out of their last 20 rounds, Rory has won 14.

(i) Based on these results, give and estimate of the probability that David will win the next round.

(ii) Given that David wins the next five rounds, give an estimate of the probability that Rory wins the next round after that.

Answers

Exercise 10.5

1. 50

2. (i) 10   (ii) 10   (iii) 30

3. (i) 160   (ii) 240

4. $\frac{3}{10}$

5. (i) $\frac{1}{2}$ (ii) $\frac{3}{4}$

(iii) Nicky's result as it involved a greater number of trials

6. (i) $\frac{3}{8}$ (ii) $\frac{27}{32}$

7. (ii) 0.6   (iii) 180

8. (i) $\frac{1}{6}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$

9. (i) 4 (relative frequency of 0.525)

(ii) 2 (relative frequency of 0.23)

Answers

10. Ben's (relatively even spread of outcomes as between the 6 faces); Joe's (relative frequency of a "6" was $\frac{1}{72}$)

11. Red – 1, White – 2, Blue – 3

12. (i) $\frac{12}{25}$ (ii) 48

13. (i) Red $-\frac{7}{50}$, Green $-\frac{12}{25}$

(ii) Red – one side, Green – three sides;

$\frac{7}{50} \simeq \frac{1}{6}, \frac{12}{25} \simeq \frac{1}{2}$

14. 100

15. (i) 10 sectors/segments with 5 coloured red, 3 white and 2 blue

16. (i) $\frac{3}{10}$ (ii) $\frac{14}{25}$