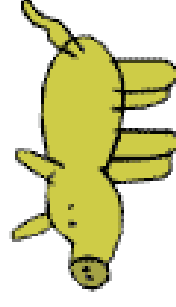


## 12. Ratio



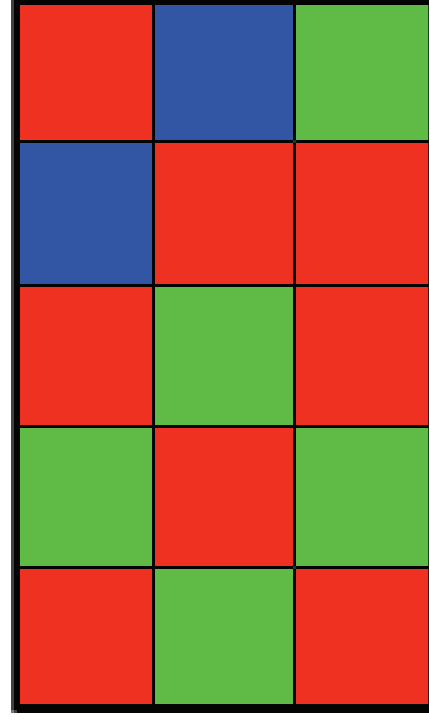
### What are Ratios and Why do we need them?...

Ratios are just a nice easy way of showing the **relative sizes of something**, whether it be **quantities of money**, **lengths of desks**, amount of time, pretty much anything you can **measure** can be expressed as a ratio.

Ratios are also very closely linked to **Fractions**, and they behave in a very similar way. **So...** if you can understand fractions, you'll be flying here!

### 1. Writing Ratios

There is a funny way of ratios that requires the use of a colon : let me show you...



The ratio of **red** squares to **green** squares is:

$$8 : 5$$

Because for every 8 red squares, there are 5 green:

The ratio of **green** squares to **red** squares is:

$$5 : 8$$

The ratio of **blue** squares to **red** squares is:

$$2 : 8$$

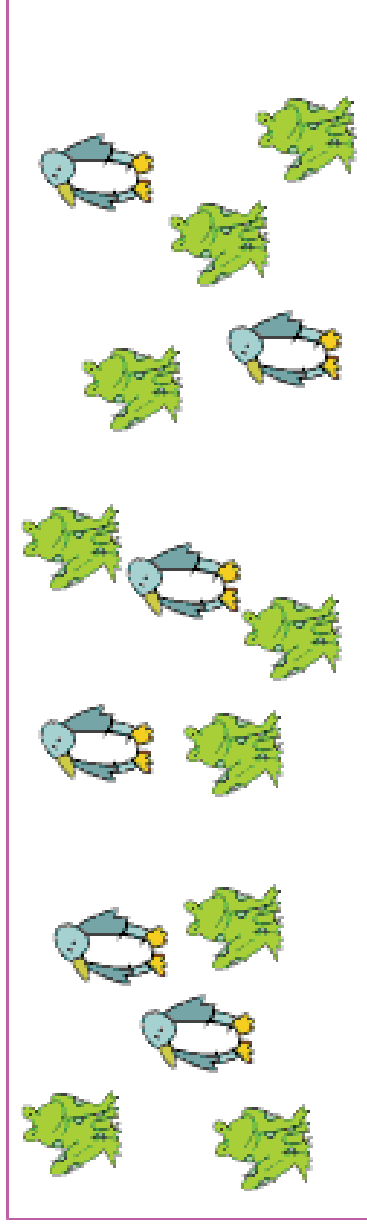
## 2. Simplifying Ratios

For the whole box, the ratio of frogs to penguins is:

$$9 : 6$$

But, can you see that for every 3 frogs, there are 2 penguins?... So, it's also:

$$3 : 2$$



### Method for Simplifying Ratios

Just like with fractions, whatever you multiply/divide one side by, make sure you do the exact same to the other side!

Keep dividing until each side has no common factors

#### Example 1 Simplify 14 : 21

Okay, we are looking for factors common to both sides... how about **7**!

Divide both sides by 7 ...

$$\div 7 \left( \begin{array}{l} 14 : 21 \\ 2 : 3 \end{array} \right) \div 7$$

Check for other common factors to make it even simpler?... **No, so we're done!**

#### Example 2 Simplify 60 : 45

Okay, we are looking for factors common to both sides... how about **15**!

Divide both sides by 15 ...

$$\div 15 \left( \begin{array}{l} 60 : 45 \\ 4 : 3 \end{array} \right) \div 15$$

Check for other common factors to make it even simpler?... **No, so we're done!**

### 3. “1 to n” and “n to 1”!

Sometimes the mean examiners aren't happy with you merely simplifying a ratio, they want it expressed as either **1 : n** or **n : 1**. Sounds hard, but so long as you can simplify ratios, and you remember that **n** is just a **number**, you'll be fine!

Example 1 Express **8 : 14** in the form **1 : n**

Right, what this question is asking you to do is to change **8 : 14** into **1 : n**, where **n** is just a number for you to find.

Now, the important thing here is that you stick to the rule: *whatever you multiply/divide one side by, do the exact same to the other side.*

We need to change the **8** into **1**, so we must **divide by... erm... 8!**

$$\begin{array}{c} \div 8 \\ \left( \begin{array}{l} 8 : 14 \\ 1 : ? \end{array} \right) \div 8 \end{array}$$

Dividing our other side by 8 gives us our final answer...

$$1 : 1.75$$

Example 1 Express **0.3 : 0.15** in the form **n : 1**

Again, we just need to change **0.3 : 0.15** into **n : 1**, sticking to our rule.

Problem: what on earth do we divide **0.15** by to give us **1**?... Well, anything divided by itself is **1**, so how about by **0.15!**

$$\begin{array}{c} \div 0.15 \\ \left( \begin{array}{l} 0.3 : 0.15 \\ ? : 1 \end{array} \right) \div 0.15 \end{array}$$

So, to get the other side, we just divide **0.3** by **0.15**, which gives us our answer...

$$2 : 1$$



## 4. Classic Ratio Questions

The types of questions on ratio that you usually get in the exam sound really nasty, but all they require is a little knowledge of what we have done before.

Remember: Whatever you multiply/divide one side by, do the same to the other!

### Example

Mr Barton has conned his Mum into making him a cake. It says on the packet that the ingredients must be mixed in the following ratios:

Flour (g)	:	Butter (g)	:	Eggs	:	Sugar (g)
400	:	220	:	3	:	25

(a) If my Mum has 1000g of flour, how much butter does she need?

(b) If she has 2 eggs, how much sugar does she need?

Always set these sort of questions out the same way – write the **original ratios on the top**, write the **new amount you know on the bottom**, and ask yourself: "what do I need to do to get from my original amount to my new amount?"

(a) This is what we've got:

$$\begin{array}{cc} \text{flour} & \text{butter} \\ 400 : 220 & \\ \times 2.5 & \left( \begin{array}{l} \times 2.5 \\ \times 2.5 \end{array} \right) \end{array}$$

$1000 \div 400 = 2.5$

How do I get from 400 to 1000?... I multiply by 2.5!, so let's do the same to the butter!

$$220 \times 2.5 = \underline{550\text{g}}$$

(b) This is what we've got:

$$\begin{array}{cc} \text{eggs} & \text{sugar} \\ 3 : 25 & \\ \times \frac{2}{3} & \left( \begin{array}{l} \times \frac{2}{3} \\ \times \frac{2}{3} \end{array} \right) \end{array}$$

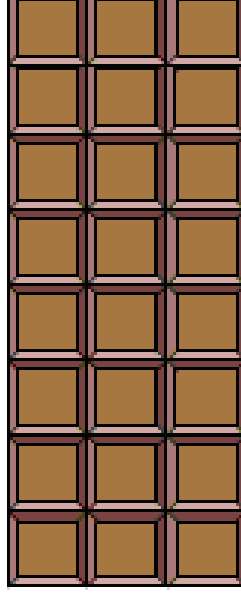
$2 \div 3 = \frac{2}{3}$

How do I get from 3 to 2?... I multiply by  $\frac{2}{3}$  so let's do the same to the sugar!

$$25 \times \frac{2}{3} = 16\frac{2}{3}\text{g}$$

## 5. Sharing in a Given Ratio

For baking me the cake, I decide to share this bar of chocolate with my Mum in the ratio **5 : 3** (I am the 5, of course). How many pieces does each of us get?



### Method for Sharing Ratios

1. Add up the **total number of parts** you are sharing between
2. Work out how much **one part** gets
3. Use this to work out how much **everybody gets!**

### Example 1

The **Chocolate Example!**

1. Okay, so I get **5** parts, and my Mum gets **3** parts, so in total there are... **8 parts!**
2. There are **24** pieces of chocolate all together, so each part must be worth...

$$24 \div 8 = \underline{3} \text{ pieces}$$

3. I have 5 parts, so I get:

$$3 \times 5 = \underline{15} \text{ pieces}$$

And Mum's 3 parts get her:

$$3 \times 3 = \underline{9} \text{ pieces}$$

$$\text{Look: } 15 + 9 = 24!$$

### Example 2

Share **£845** in the ratio **8 : 3 : 2**

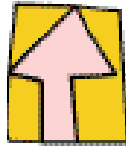
1. Okay, so in total there are ... **13 parts!**
2. We have **£845** to share, so each part receives ...  $845 \div 13 = \underline{£65}$
3. How much does each person get? ...

8 parts	$65 \times 8 = \underline{£520}$
---------	----------------------------------

3 parts	$65 \times 3 = \underline{£195}$
---------	----------------------------------

2 parts	$65 \times 2 = \underline{£130}$
---------	----------------------------------

Look: $520 + 195 + 130 = \underline{£845!}$
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## 13. Proportion



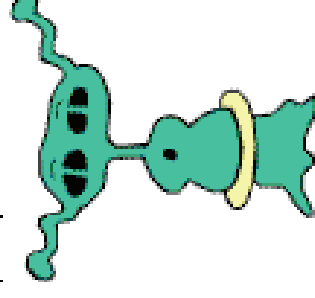
### What does proportion mean, and what's that funny fish symbol?

If two variables are proportional to each other, it just means that they are related to each other in a specific way.

The funny fish symbol  $\propto$  just means "is proportional to"

### 1. Two types of Proportion

Again, how many of these you need to worry about depends on your maths set, and your exam board, and stuff like that, but here are the two main types of proportion:

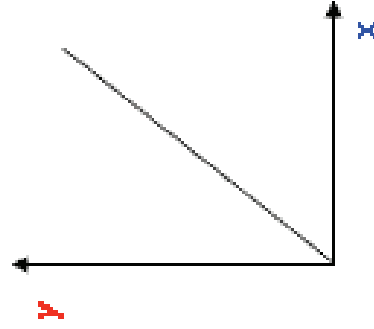


#### (a) Direct Proportion

Both variables increase or decrease together

#### (i) Linear

##### Graph



##### Fancy Lingo

$$y \propto x$$

y is proportional to x

y is directly proportional to x

y varies directly as x varies

##### Example

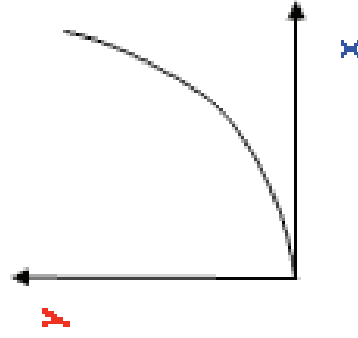
x could be the number of KitKat Chunkys that you buy

y could be the total cost of those KitKat Chunkys

As the number you buy increases, so too does the total cost

## (ii) Quadratic

Graph



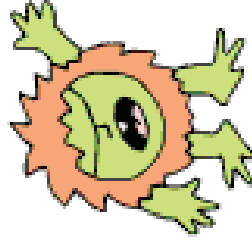
Fancy Lingq

$$y \propto x^2$$

y is proportional to  $x^2$

y is directly proportional to  $x^2$

y varies directly as  $x^2$  varies



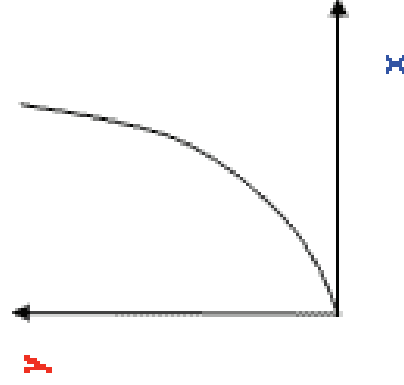
Example

x could be the amount of money you spend advertising a gig

y could be the number of people who turn up to the gig

As the amount of advertising increases, word of mouth quickly spreads, and the number of people who go to the gig goes up by a lot.

Graph



Fancy Lingq

$$y \propto x^3$$

y is proportional to  $x^3$

y is directly proportional to  $x^3$

y varies directly as  $x^3$  varies

Example

x could be the amount of time you spend on [mrbartonmaths.com](http://mrbartonmaths.com)

y could be your maths exam mark

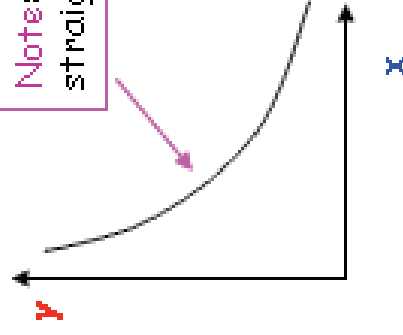
As the amount of time you spend revising on the site increases, everything begins to fall into place, and your marks just get higher and higher with each extra minute!

## (b) Inverse Proportion

As one variable goes up, the other goes down

### (i) Inverse

Graph



Note: Not a straight line!

Fancy Lingq

$$y \propto 1/x$$

y is inversely proportional to x

Example

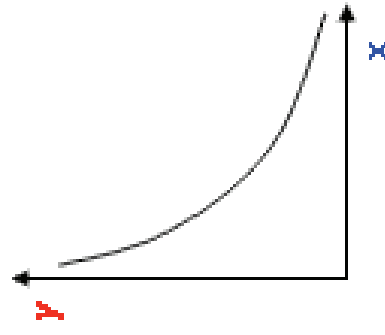
x could be the number of people you convince to join you on a road trip

y could be the amount each person must pay for petrol

As the number of people in the car increases, the amount everyone has to pay falls

### (ii) Quadratic Inverse

Graph



Fancy Lingq

$$y \propto 1/x^2$$

y is inversely proportional to  $x^2$

Example

x could be the number of hours you spend watching Big Brother

y could be your number of brain cells

As the hours increase, your brain cells disappear and an increasing rate!