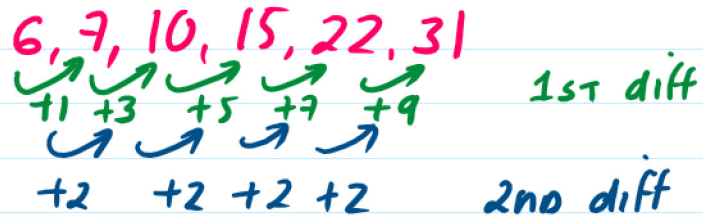


A quadratic Sequence will be in the form

$$T_n = an^2 + bn + c, \text{ where } a, b, c \in \mathbb{Z}$$

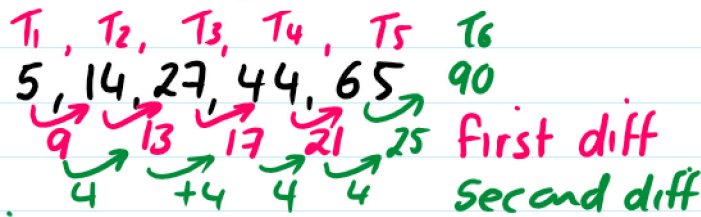
A quadratic sequence will have a **second difference**

Eg 1) The sequence is  
find the 1st and 2nd  
difference



The second difference is constant  $\therefore$  the sequence is quadratic

Eg 2) Show the sequence 5, 14, 27, 44, 65 is quadratic then find the 6<sup>th</sup> term in the sequence.



2nd diff is constant  $\therefore$  quadratic

Classwork Pg 297 Q1-4



**PROJECT MATHS**

# Text & Tests

**Leaving 3 Certificate**

*chapter*

**10**

## **Patterns and Sequences**

**Section 10.7 Quadratic sequences**

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### Exercise 10.7

2. Which of these sequences are quadratic?

(i) 6, 8, 12, 18, 26, 36, ... *yes*

(iii) 3, 4, 7, 12, 19, 28, ... *yes*

$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 9$   
 $2 \rightarrow 4 \rightarrow 6 \rightarrow 8$   
 $3 \rightarrow 6 \rightarrow 9$

(ii) 6, 8, 10, 12, 14, 16, ... *linear (No)*

(iv) 0, 3, 8, 15, 24, ... *yes*

$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 9$   
 $2 \rightarrow 4 \rightarrow 6 \rightarrow 8$   
 $3 \rightarrow 6 \rightarrow 9$

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### Exercise 10.7

3. Find the first 5 terms of the sequences with these  $n$ th terms:

(i)  $T_n = n^2 + 4$

$$T_1 = (1)^2 + 4 = 5$$

$$T_2 = (2)^2 + 4 = 8$$

$$T_3 = (3)^2 + 4 = 13$$

$$T_4 = (4)^2 + 4 = 20$$

$$T_5 = (5)^2 + 4 = 29$$

(ii)  $T_n = n^2 - 1$

$$T_1 = (1)^2 - 1 = 0$$

$$T_2 = (2)^2 - 1 = 3$$

$$T_3 = (3)^2 - 1 = 8$$

$$T_4 = (4)^2 - 1 = 15$$

$$T_5 = (5)^2 - 1 = 24$$

(iii)  $T_n = 2n^2 + n + 1$

$$T_1 = 2(1)^2 + (1) + 1 = 4$$

$$T_2 = 2(2)^2 + (2) + 1 = 10$$

$$T_3 = 2(3)^2 + (3) + 1 = 20$$

$$T_4 = 2(4)^2 + (4) + 1 = 33$$

$$T_5 = 2(5)^2 + (5) + 1 = 51$$

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### Exercise 10.7

4. Find the 10th term of the sequence with  $T_n = n^2 + 2n - 4$ .

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### Exercise 10.7

5. Write the sequence 4, 7, 12, 19, 28, ... as follows

4	7	12	19	28
-	-	-	-	-
-	-	-	-	-

first difference

second difference

If  $T_n = an^2 + bn + c$ , use the second difference to write down the value of  $a$ .

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### Exercise 10.7

6. Find an expression for the  $n$ th term of each of these quadratic sequences:

(i) 5, 8, 13, 20, 29, ...

(ii) 2, 8, 18, 32, 50, ...

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### Exercise 10.7

7. Find an expression for the  $n$ th term of the sequence

7, 10, 15, 22, 31, ...

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### Exercise 10.7

8. Show that the  $n$ th term of the quadratic sequence 8, 15, 26, 41, 60, ... is  $2n^2 + n + 5$ .

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### Exercise 10.7

9. Use a difference table to work out the  $n$ th term of this sequence:  
3, 8, 15, 24, 35, ...

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### Exercise 10.7

10. Each layer of cubes in these designs is a square.

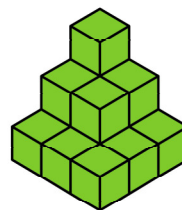
- (i) How many cubes will be in Model 4?  
 (ii) Use the pattern to write down the number of cubes in Model 5.



Model 1



Model 2



Model 3

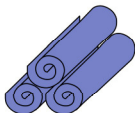
- (iii) Show that the expression  $\frac{n}{6}(n+1)(2n+1)$  gives the correct number of cubes in Model 3 and Model 4.  
 (iv) Use the expression for the  $n$ th term to find how many cubes in Model 10.

### Exercise 10.7

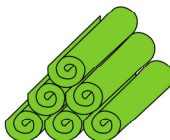
11. Here are some rugs stacked in a carpet showroom.



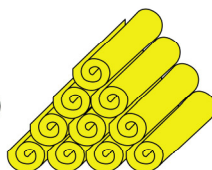
Stack 1



Stack 2



Stack 3



Stack 4

- (i) Copy and complete the table on the right.  
 (ii) Draw a difference table for the sequence for the number of rugs.  
 (iii) Use the differences to find an expression for the  $n$ th term of this sequence.  
 (iv) Use the  $n$ th term to find the number of rugs in Stack 20.

<b>Stack number</b>	1	2	3	4	5	...
<b>Number of rugs</b>	1	3				...



**Answers 10.7**

1. (i) 18, 24      (ii) 38, 51      (iii) 47, 62
2. (i) Yes      (ii) No      (iii) Yes      (iv) Yes
3. (i) 5, 8, 13, 20, 29      (ii) 0, 3, 8, 15, 24  
(iii) 4, 11, 22, 37, 56
4. 116
5.  $4 \quad 7 \quad 12 \quad 19 \quad 28; a = 1$   
 $\quad 3 \quad 5 \quad 7 \quad 9$   
 $\quad 2 \quad 2 \quad 2$
6. (i)  $T_n = n^2 + 4$       (ii)  $T_n = 2n^2$
7.  $T_n = n^2 + 6$
9.  $T_n = n^2 + 2n$
10. (i) 30      (ii) 55  
(iv) 385
11. (iii)  $T_n = \frac{n^2 + n}{2}$       (iv) 210