Problems with more than one shape

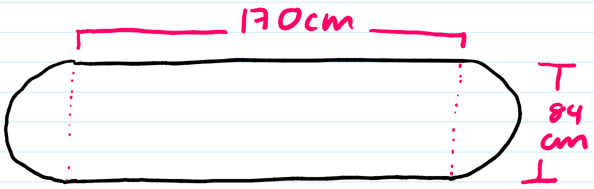
07 December 2019 09:49

Take care reading these Questions

Highlight key information

Give a unit in your answer – Area = units^2 Volume = units^3

Eg 1) A capsule is made up of a cylindrical section and two hemispherical ends.



1) Find the surface area of the capsule in cm^2
Give your answer correct to two significant figures

2) Find the volume of the capsule in m^3.
Give your answer correct to two decimal places?

Solution

1) Surface Area

$$\text{radius} = \frac{\text{Diameter}}{2}$$

$$\Rightarrow r = \frac{84}{2} = 42\text{cm}$$

The two hemispherical ends make 1 sphere

Surface area of a sphere $= 4\pi r^2$

$$= 4 \times \pi \times 42^2$$

$$= 7056\pi \text{ or } 22167.08$$

Cylindrical section

Surface area of a cylinder $= 2\pi r h$

$r = 42$ $h = 170$

$$= 2\pi(42)(170)$$

$$= 14280\pi \text{ or } 44861.94$$

Total surface area = Cylinder + Sphere.

$$= 44861.94 + 22167.08$$

$$= 67029.02$$

Ans to Two significant figures 67000cm^2

$= 67029.02$

Ans to Two significant figures $67000 cm^2$

2) Volume

Volume of sphere + volume of cylinder

formula $4/3 \pi r^3 \quad + \quad \pi r^2 h$

$4/3 \times \pi \times 42^3 \quad + \quad \pi \times 42^2 \times 170$

$98784\pi \quad + \quad 299880\pi$

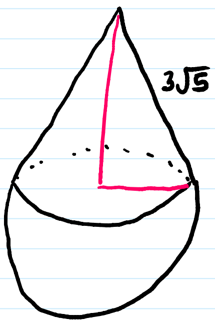
or $310339.09 \quad + \quad 942100.81$ Add together

$= 1252439.895 cm^3$

Convert to meters → divide by $(100)^3$

$1252439.895 \div (100)^3 = 1.25 m^3$

Q2) A solid is the shape of a hemisphere surmounted by a cone.



1) The volume of the hemisphere is $18\pi cm^3$. Find the radius of the hemisphere

2) The length of the slant of the cone is $3\sqrt{5} cm$. Find the vertical height of the cone.

3) Show the volume of the cone is equal to the volume of the hemisphere.

4) This solid is melted down and recast in the shape of a solid cylinder. The height of the cylinder is 9cm. Find its radius.

Solution

1) Formula Volume of a hemisphere

$V = 2/3 \pi r^3$ Vol of hemisphere $= 18\pi$

$\therefore \frac{2}{3} \pi r^3 = 18\pi$ Cancel the π ($\div$ by π)

$= \frac{2}{3} r^3 = 18$ Divide both sides by $2/3$

$= \frac{2}{3} r^3 = 18$ Divide both sides by $2/3$ (invert and multiply)

$= r^3 = 18 \times \frac{3}{2}$

$\sqrt[3]{\ }$ both sides

$= r = \sqrt[3]{27}$

$r = 3$ cm

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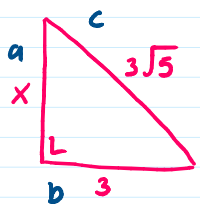
2)



Part (1) $r=3$

Find the height

Pythagoras $c^2 = a^2 + b^2$

$(3\sqrt{5})^2 = (x)^2 + (3)^2$

$45 = x^2 + 9$

$-9 \mid \sqrt{\ }$ $\quad 36 = x^2 \quad$ $\mid -9 \ \sqrt{\ }$

$\sqrt{36} = x$

$6 = x$

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Part 3) We know the hemisphere has a volume of 18π

we need to the volume of the cone.

Formula = $\frac{1}{3}\pi r^2 h$

Part (1) $r=3$ } Sub in to formula $\Rightarrow \frac{1}{3}\pi(3)^2(6)$

Part (2) $h=6$

$= 18\pi$ cm^3

The two shapes have the same volume.

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Part 4) Vol of cone + vol of hemisphere

$18\pi + 18\pi = 36\pi$ cm^3

The new cylinder was made from the original shape it has the same volume 36π cm^3

Volume of cylinder $= \pi r^2 h$

Volume of cylinder = $\pi r^2 h$

$h = 9cm$
Vol = 36π
$r = ?$

$\Rightarrow$ sub into formula $\Rightarrow$ $\pi(r)^2(9) = 36\pi$ cancel π

$r^2(9) = 36$ $\div$ by 9

$r^2 = \frac{36}{9}$

$\sqrt{\ }$ | $r^2 = 4$ | $\sqrt{\ }$

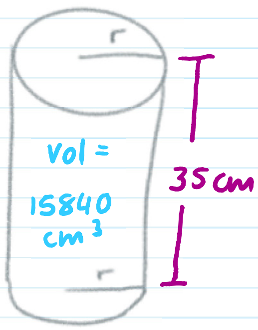
$r = \sqrt{4}$

$r = 2\,cm$

Q3) The volume of a cylinder is $15840\,cm^3$. If the height is 35cm, find the length of the radius of its base.

Solution:

Sketch



Vol = 15840
h = 35
r = ?

Formula = $\pi r^2 h$ sub values in

$\pi (r^2)(35) = 15840$

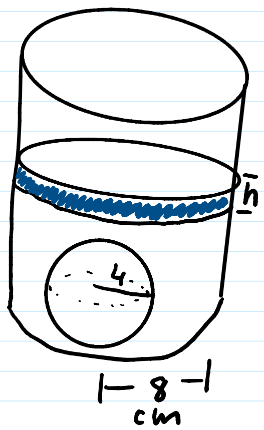
$35\pi(r^2) = 15840$

35π | $r^2 = 144$ | 35π

$\sqrt{\ }$ | $r = \sqrt{144}$ | $\sqrt{\ }$

$r = 12\,cm$

Q4) A sphere of radius 4cm is dropped into a cylinder partly filled with water. When the sphere is fully submerged the level of the water rises h cm. If the radius of the cylinder is 8cm, find the value of h.



Solution:

The volume of the sphere is equal to the shaded cylinder of height h.
(The amount the water level increased by).

Vol of sphere = vol of cylinder

formula: $4/3\,\pi r^3 = \pi r^2 h$

$r = 4$ $\quad r = 8$ sub values into formula.

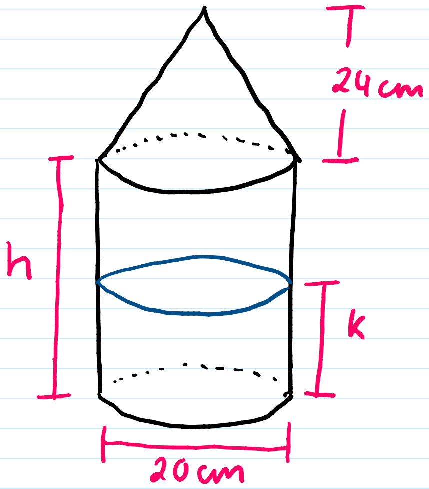
$4/3\,\pi(4)^3 = \pi(8)^2(h)$

$\div\pi$ | $4/3(64\pi) = 64\pi h$ | $\div\pi$ To cancel π

$\div 64$ | $4/3 = h$ | $\div 64$

$h = 4/3$ or 1.3 or $1\,\tfrac{1}{3}$ cm.

Q5) The diagram shows the model of a rocket ship. It's made from a solid cylinder with a solid cone on top.



i) Calculate the volume of the cone in terms of π

ii) Find the height of the cylinder if the volume of the cylinder is 4 times the volume of the cone.

iii) Find the value of K, if the volume of the cylinder of height k is half the value of the whole solid model.

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Solution

1) Vol of the cone

Formula $\frac{1}{3}\pi r^2 h$

$r = 10\text{cm}$ $h = 24$

$\frac{1}{3}\pi(10)^2(24) = 800\pi \text{ cm}^3$

2) vol of cylinder $= 4(800\pi) = 3200\pi \text{ cm}^3$

$\pi r^2 h$

$\Rightarrow \pi \times 10^2 \times h = 3200\pi$ cancel π

$\div 100$ | $= 100h = 3200$ | $\div 100$

$h = 32 \text{ cm}$

3) Volume of whole solid = Vol of cone + Vol of cylinder

$800\pi + 3200\pi$

$$= 4000\pi$$

Volume of cylinder of height k = 1/2 vol of whole solid

$$\pi r^2 k = 1/2\,(4000\pi)$$

$$\pi (10)^2 k = 2000\pi$$ ÷ π to cancel

$$100k = 2000$$

÷100 | $k = 20$cm | ÷100