

The patterns are made of matchsticks



Draw the next two terms in the pattern

T<sub>4</sub>                      T<sub>5</sub>



Copy and complete the table

Pattern Number	Number of matchsticks
1	3
2	5
3	7
4	9
5	11

Investigate if the number of matchsticks is an arithmetic pattern

arithmetic pattern - linear  
will go up or down by a constant amount each term.

T<sub>1</sub> T<sub>2</sub> T<sub>3</sub> T<sub>4</sub> T<sub>5</sub>  
3, 5, 7, 9, 11  
↘ ↗ ↘ ↗ ↘ ↗  
+2 +2 +2 +2

d = common difference  
d = 2

Find the n<sup>th</sup> term rule for the number of matchsticks

- ① a = 3
  - ② d = 2
  - ④ T<sub>n</sub> = 3 + (n-1)2
- 3 + 2n - 2

Ans = T<sub>n</sub> = 2n + 1

Q1) Find the number of matchsticks in the 21<sup>st</sup> pattern T<sub>21</sub> n=21

Q2) Which pattern contains 73 matchsticks? n=?

Solution: T<sub>n</sub> = 2n + 1

Q1

n = 21 ⇒ sub in for n

$$T_{21} = 2(21) + 1$$

$$42 + 1$$

$$= 43 \text{ matchsticks}$$

Q2

$$T_n = 2n + 1$$

Find n

$$2n + 1 = 73 \quad \text{solve for n}$$

$$\begin{array}{r|l} -1 & 2n = 72 & -1 \\ \hline \div 2 & n = 36 & \div 2 \end{array}$$

**PROJECT MATHS**  
**Text & Tests**  
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chapter

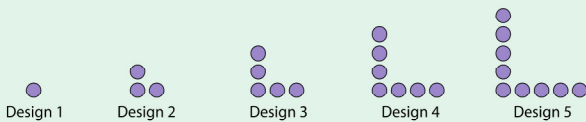
10

**Patterns and Sequences**

**Section 10.3 Sequences from shapes**

**Example 1**

These designs are made by arranging counters in L-shapes.



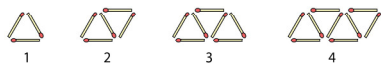
- (i) Copy and complete this table for these designs.

<b>Design number</b>	1	2	3	4	5
<b>Number of counters</b>	1				

- (ii) How many counters are in the 6th design?  
 (iii) How many counters are needed to make the 15th design?  
 Explain how you worked out your answer.  
 (iv) Which design uses 99 counters?  
 (v) Is it possible to make one of these designs with exactly 40 counters.  
 Explain your answer.

Exercise 10.3

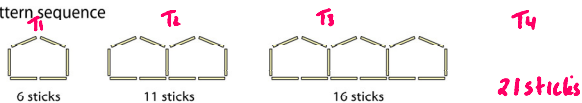
1. A pattern of triangles is built up from matchsticks.



- (i) Draw the 5th set of triangles in this pattern.
- (ii) Write down the sequence of numbers generated by the matchsticks in the first six patterns.
- (iii) Find an expression in  $n$  for the number of matches in the  $n$ th set of triangles.
- (iv) How many matches are needed for the 50th set of triangles?

Exercise 10.3

2. Here is a pattern sequence



- (i) Draw the 4th pattern in this sequence. *6, 11, 16, 21, 26, 31*
- (ii) Write down the sequence of numbers generated by the sticks in the first six patterns.
- (iii) Show that the number of sticks in the  $n$ th pattern is given by  $T_n = 5n + 1$ .
- (iv) How many sticks are required for the 20th pattern?
- (v) For which pattern are 51 sticks required?

*Handwritten solutions:*

- ①  $a = 6$
- ②  $d = 5$
- ④  $T_n = 6 + (n-1)5$   
 $6 + 5n - 5$   
 $T_n = 5n + 1$

*Handwritten solutions:*

iv)  $T_{20} = 5(20) + 1$   
 $100 + 1$   
 $T_{20} = 101$  matchsticks

*Handwritten solutions:*

v)  $n = ?$   $5n + 1 = 51$   
 $-1 \quad | \quad 5n = 50 \quad | \quad -1$   
 $\div 5 \quad | \quad n = 10 \quad | \quad \div 5$   
 $T_{10} = 51$

Exercise 10.3

3. Complete the table of values for this sequence of matchstick patterns.



Number of squares	1	2	3	4	5	6
Number of matchsticks	4	7	10	13	16	19

- (i) How many matchsticks are required for the 6th pattern?
- (ii) Find an expression in  $n$  for the  $n$ th pattern.
- (iii) Use the expression found to find the number of matchsticks required for the 50th pattern.

*Handwritten solutions:*

ii)  $n^{\text{th}}$  term

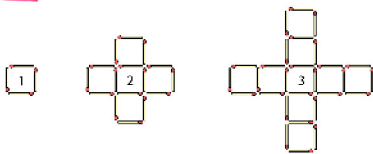
- ①  $a = 4$
- ②  $d = 3$
- ④  $T_n = 4 + (n-1)3$   
 $4 + 3n - 3$   
 $T_n = 3n + 1$

*Handwritten solutions:*

iii)  $T_{50} = 50^{\text{th}}$  pattern  
 $T_{50} = 3(50) + 1$   
 $150 + 1$   
 $T_{50} = 151$  matchsticks.

Exercise 10.3

4. A pattern of squares is built up from matchsticks as shown.



- (i) Draw the 4th pattern.
- (ii) Find an expression in  $n$  for the number of squares in the  $n$ th pattern.
- (iii) How many squares are there in the 30th pattern?
- (iv) Which pattern contains exactly 77 squares?

ii)  $n^{\text{th}}$  term rule

①  $\Rightarrow a = 1$

②  $d = 4$

④  $T_n = 1 + (n-1)4$   
 $1 + 4n - 4$   
 $T_n = 4n - 3$

$T_n$   
 $4n - 3 = 77$   
 $+3 \quad +3$   
 $\div 4 \quad \div 4$   
 $4n = 80$   
 $n = 20$

iii  $n = 30$

$T_{30} = 4(30) - 3$   
 $120 - 3$   
 $T_{30} = 117$

Exercise 10.3

5. Look at these matchstick shapes.



- (i) Copy and complete the table below:

Shape number	1	2	3	4	5
Number of matchsticks	5	9	...	...	...

- (ii) How many matchsticks are there in Shape 77?
- (iii) Find an expression for the number of matchsticks in Shape  $n$ .
- (iv) Which shape contains exactly 101 matchsticks?

Exercise 10.3

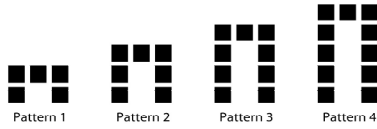
- 6. (i) Find the eighth term of the sequence whose  $n$ th term is  $4n - 1$ .
- (ii) Find the  $n$ th term of the sequence whose first four terms are

2 8 14 20

**Exercise 10.3**

7. Each of these patterns uses black tiles.

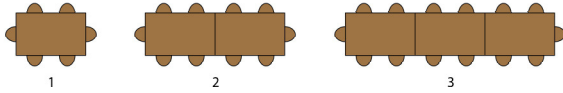
- (i) How many black tiles will be in pattern 5?
- (ii) How many black tiles will be in pattern 10?



- (iii) Find an expression for the number of black tiles in pattern  $n$ .
- (iv) How many tiles will be in pattern 100?
- (v) Which pattern will have exactly 101 tiles?

**Exercise 10.3**

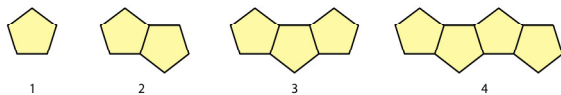
8. A conference centre had tables each of which could sit six people. When put together, the tables could seat people as shown.



- (i) How many people could be seated at 4 tables?
- (ii) How many people could be seated at  $n$  tables put together in this way?
- (iii) A conference had 90 people who wished to use the tables in this way. How many tables would they need?

**Exercise 10.3**

9. Regular pentagons of side length 1 cm are joined together to make a pattern as shown.



- (i) Write down the perimeter of each of the first 4 shapes. (Do not include internal lines.)
- (ii) What is the perimeter of the 5th and 6th shapes?
- (iii) Find an expression for the perimeter of the  $n$ th shape.
- (iv) Find the length of the perimeter of the 50th shape.
- (v) Which shape has a perimeter of length 92 cm?

Answers 10.3

1. (i)  (ii) 3, 5, 7, 9, 11, 13

(iii)  $2n + 1$

(iv) 101



(ii) 6, 11, 16, 21, 26, 31

(iv) 101

(v)  $T_{10}$

3.

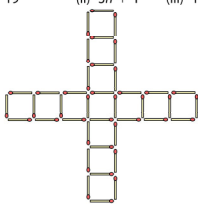
No. of squares	1	2	3	4	5
No. of m'sticks	4	7	10	13	16

(i) 19

(ii)  $3n + 1$

(iii) 151

4. (i)



(ii)  $4n - 3$

(iii) 117

(iv) 20th

5. (i)

Shape no.	1	2	3	4	5
No. of m'sticks	5	9	13	17	21

(ii) 29

(iii)  $4n + 1$

(iv) Shape 25

6. (i) 31

(ii)  $6n - 4$

7. (i) 13

(ii) 23

(iii)  $2n + 3$

(iv) 203

(v) Pattern 49

8. (i) 18

(ii)  $4n + 2$

(iii) 22

9. (i) 5 cm, 8 cm, 11 cm, 14 cm

(ii) 17 cm, 20 cm

(iii)  $3n + 2$

(iv) 152 cm

(v) 30th