

A pattern is made up of terms  
Eg 1  $T_1$   $T_2$   $T_3$   $T_4$   $T_5$   
2, 5, 8, 11, 14  
Arrows between terms are labeled with 3.

Describe in words : The pattern starts at 2 and 3 is added each time

start is the first term

The second term =  $T_2 = 5$

The difference between each term is 3

The difference is constant its a straight line when graphed

The  $n^{th}$  term of a sequence

Log Tables Pg 22

$T_n = a + (n-1)d$  where  $a$  = first term and  $d$  = common difference.

A sequence  $T_1$   $T_2$   $T_3$   $T_4$   $T_5$   $T_6$   $T_7$   $T_8$   
7, 4, 1, -2, -5, -8, -11, -14  
Arrows between terms are labeled with -3.  
Prove it is an arithmetic (linear) pattern and find the next two terms

Common difference is -3

When given the  $n^{th}$  term rule for the pattern and asked to find the pattern

- 1) sub in values into  $n$
- 2) start with 1

Eg find the first three terms in the sequence when  $T_n = n + 4$

$T_1^n = (1^n) + 4 = 5$       Pattern 5, 6, 7  
 $T_2^n = (2^n) + 4 = 6$        $\downarrow$   $\downarrow$   
 $T_3^n = (3^n) + 4 = 7$       +1 +1

$T_{100} = (100) + 4 = 104$

Classwork Pg 280 Q1(ii)  $\rightarrow$  (iv)  
Q2  $\rightarrow$  6.

**PROJECT MATHS**

# Text & Tests

Leaving **3** Certificate

chapter

**10**

## Patterns and Sequences

### Section 10.2 The $n$ th term of a sequence

#### Example 1

The  $n$ th term of a sequence is  $4n - 3$ .  
Write down the first five terms of the sequence.

### Example 2

Find the  $n$ th term of the sequence 3, 7, 11, 15, ...

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### Example 3

Find the  $n$ th term and the 20th term of the sequence 4, 7, 10, 13, 16, ...

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### Exercise 10.2

1. Use each of the following rules to write down the first three terms of the sequence:

(i)  $n + 4$  for  $n = 1, 2, 3$

(ii)  $2n + 1$  for  $n = 1, 2, 3$

(iii)  $4n + 3$  for  $n = 1, 2, 3 = 7, 11, 15$

(iv)  $3n - 2$  for  $n = 1, 2, 3$ .

$$T_n = 2n + 1$$

$$T_1 = 2(1) + 1 = 3$$

$$T_2 = 2(2) + 1 = 5$$

$$T_3 = 2(3) + 1 = 7$$

$$T_n = 3n - 2$$

$$T_1 = 3(1) - 2 = 1$$

$$T_2 = 3(2) - 2 = 4$$

$$T_3 = 3(3) - 2 = 7$$

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### Exercise 10.2

2. In each of the following sequences the  $n$ th term,  $T_n$ , is given.

Write down the first four terms of each sequence:

(i)  $T_n = 3n$

(ii)  $T_n = 2n + 3$

(iii)  $T_n = 3n - 2$

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### Exercise 10.2

3. If  $T_n = 3n - 4$ , find  $T_1$ ,  $T_3$  and  $T_{10}$ .

$$T_1 = 3(1) - 4 = -1$$

$$T_3 = 3(3) - 4 = 5$$

$$T_{10} = 3(10) - 4 = 26$$

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### Exercise 10.2

4. Write down the first three terms of each of these sequences where the  $n$ th term is given.

(i)  $T_n = n^2$

(ii)  $T_n = n^2 + 3$

(iii)  $T_n = 2n^2 + 1$

$$T_1 = (1)^2 = 1$$

$$T_2 = (2)^2 = 4$$

$$T_3 = (3)^2 = 9$$

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Exercise 10.2

5. If  $T_n = 2n^2 - 1$ , work out

- (i)  $T_1$       (ii)  $T_2$       (iii)  $T_5$       (iv)  $T_{10}$

$$T_1 = 2(1)^2 - 1 = 1 \quad T_2 = 2(2)^2 - 1 = 7 \quad T_5 = 2(5)^2 - 1 = 49 \quad T_{10} = 2(10)^2 - 1 = 199$$

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Exercise 10.2

6. If  $T_n = 2n - 6$ , show that  $T_1 + T_5 = 0$ .

$$T_1 = 2(1) - 6 = -4 \quad T_5 = 2(5) - 6 = +4$$

$$T_1 + T_5 = 0 \\ -4 + 4 = 0 \checkmark$$

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Exercise 10.2

7. Find the next two terms and the  $n$ th term of each of the following sequences:

- (i) 3, 5, 7, 9, 11, ...      (ii) 4, 7, 10, 13, ...  
 (iii) 2, 6, 10, 14, 18, ...      (iv) 5, 9, 13, 17, ...

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Exercise 10.2

8. Find the  $n$ th term and the 20th term of each of these sequences:

- (i) 2, 5, 8, 11, ...      (ii) 6, 8, 10, 12, ...  
 (iii) 4, 9, 14, 19, ...      (iv) 2, 7, 12, 17, ...

② When given a pattern and asked to find the  $n$ th term rule ( $T_n$ )

Method

- 1) Find the first term ( $a$ )
- 2) Find the common difference ( $d$ )
- 3) Sub the values for  $a$  and  $d$  into the formula for  $T_n$
- 4) Formula for  $T_n = a + (n-1)d$  pg 22 log tables.

Eg 1) Find the  $n$ th term rule for the pattern

$$\begin{array}{cccccc} T_1 & T_2 & T_3 & T_4 & T_5 \\ 3, & 5, & 7, & 9, & 11 \\ \uparrow & \uparrow & \uparrow & \uparrow & \\ +2 & +2 & +2 & +2 & \end{array}$$

$$T_1 = a = 3$$

Common difference  $d = +2$  } sub into formula

$$T_n = a + (n-1)d$$

$$T_n = 3 + (n-1)2$$

↖ multiply

$$T_n = 3 + 2n - 2 \quad \text{Add like terms}$$

$$T_n = 2n + 1$$

Eg 2) Find the  $n$ th term rule of the pattern

$$\begin{array}{cccc} T_1 & T_2 & T_3 & T_4 \\ 4, & 7, & 10, & 13 \\ \uparrow & \uparrow & \uparrow & \\ +3 & +3 & +3 & \end{array}$$

$$T_1 \text{ (first term)} \Rightarrow a = 4$$

Common difference  $\Rightarrow d = 3$  } Sub into formula  $T_n = a + (n-1)d$

$$T_n = 4 + (n-1)3$$

↖ multiply

$$T_n = 4 + 3n - 3$$

$$T_n = 3n + 1 \quad n^{\text{th}} \text{ term rule.}$$

Pg 280 Q 7 (iii)  $\rightarrow$  (iv) CLASS WORK

Exercise 10.2

8. Find the  $n$ th term and the 20th term of each of these sequences:

(i) 2, 5, 8, 11, ...

(ii) 6, 8, 10, 12, ...

(iii) 4, 9, 14, 19, ...

(iv) 2, 7, 12, 17, ...

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Exercise 10.2

9. For the sequence 6, 11, 16, 21, ...

find (i)  $T_5$  (ii)  $T_{20}$  (iii)  $T_{100}$

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Exercise 10.2

10. The  $n$ th term of a sequence is given by  $T_n = 3n - 4$ .

Which term of the sequence is 23?

[Hint: Let  $3n - 4 = 23$ .]

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Answers 10.2

1. (i) 5, 6, 7                      (ii) 3, 5, 7  
       (iii) 7, 11, 15              (iv) 1, 4, 7
2. (i) 3, 6, 9, 12    (ii) 5, 7, 9, 11    (iii) 1, 4, 7, 10
3. -1, 5, 26
4. (i) 1, 4, 9            (ii) 4, 7, 12        (iii) 3, 9, 19
5. (i) 1                  (ii) 7                (iii) 49            (iv) 199
7. (i) 13, 15;  $2n + 1$     (ii) 16, 19;  $3n + 1$   
       (iii) 22, 26;  $4n - 2$     (iv) 21, 25;  $4n + 1$
8. (i)  $T_n = 3n - 1, T_{20} = 59$   
       (ii)  $T_n = 2n + 4, T_{20} = 44$   
       (iii)  $T_n = 5n - 1, T_{20} = 99$   
       (iv)  $T_n = 5n - 3, T_{20} = 97$
9. (i)  $5n + 1$         (ii) 101            (iii) 501
10.  $T_9$