T&T2 11.1 Revision of formulae

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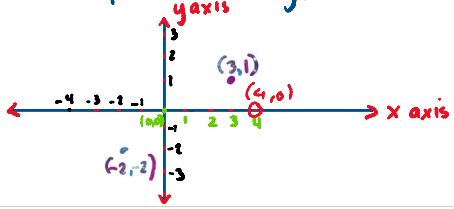
T&T2 11.1 Revision of...

chapter 11 **Coordinate Geometry – The Line**

Section 11.1 Revision of formulae

Points in the plane (x, y) $(0,0)$ - origin

Plane



To find a point go out ⟷ on the x axis (horizontal)
Then go up or down on the y axis ↕ vertical

(x, y)

NOTE:

① All along the x axis $y = 0$ Any point $(x, 0)$

② All along the y axis $x = 0$ Any point $(0, y)$

$A(x_1, y_1) = (1, 2)$ and $B(x_2, y_2) = (4, 5)$

$|AB|$ = Length or Distance

Formula pg 18 Log tables

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Method

① Label the two points (x_1, y_1) (x_2, y_2)

② Sub the values into the formula.

$$\sqrt{(4-1)^2 + (5-2)^2}$$

$$= \sqrt{(3)^2 + (3)^2}$$

$$\sqrt{9+9} = \sqrt{18} \quad \text{surd form}$$

$$= 3\sqrt{2} \quad [SD]$$

$$= 4.2 \quad 1 \text{ d.p}$$

H/W

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Q5

$\sqrt{18} = \sqrt{9 \times 2}$

$3\sqrt{2}$

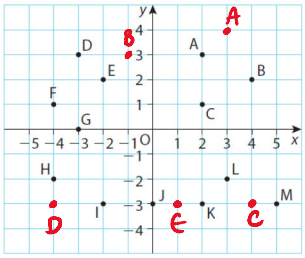
Example 1

If A(−1, 3) and B(5, 7) are two points in the plane, find

(i) |AB| (ii) the midpoint of [AB].

Exercise 11.1

1. Write down the coordinates of each of the points marked in the coordinated plane on the right:



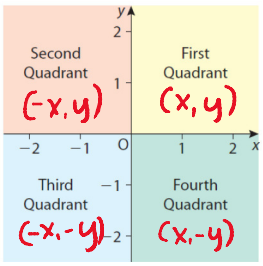
A = (2, 3)
B = (4, 2)
C = (2, 1)
D = (−3, 3)
E = (−2, 2)
F = (−4, 1)
G = (−3, 0)
H = (−4, −2)
I = (−2, −3)
J = (0, −3)
K = (2, −3)
L = (3, −2)
M = (5, −3)

2. Draw a coordinated plane from −5 to 5 on the *x*-axis and from −4 to 4 on the *y*-axis. Now plot each of the following points:

(i) A(3, 4) (ii) B(−1, 3) (iii) C(4, −3) (iv) D(−4, −3) (v) E(1, −3)

3. The four quadrants are shown on the right.
In which quadrant does each of the following points lie?

(i) (3, 5) 1st
(ii) (−2, −3) 3rd
(iii) (1, −4) 4th
(iv) (−3, 1) 2nd
(v) (3, −3) 4th
(vi) (−1, −3). 3rd



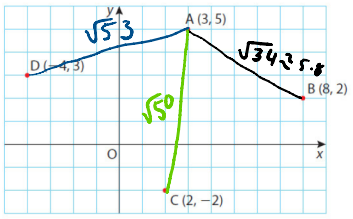
4. On which axis does each of the following points lie?

(i) (4, 0) (x, 0) x axis (ii) (−3, 0) x (iii) (0, 4) y (iv) (0, −3) y (v) (0, 0). origin

5. The points A, B, C and D are shown.

Find (i) |AB| (3,5) (8,2)
(ii) |AC| (3,5) (2,−2)
(iii) |AD|. (3,5) (−4,3)

Is |DC| = |BC|?



i) $\sqrt{(8-3)^2+(2-5)^2}$
$\sqrt{(5)^2+(-3)^2}$
$\sqrt{25+9}$
$\sqrt{34}$

ii) $\sqrt{(2-3)^2+(-2-5)^2}$
$\sqrt{(-1)^2+(-7)^2}$
$\sqrt{1+49}$
$\sqrt{50}$ $\sqrt{2\times25}$ $5\sqrt{2}$

iii) $\sqrt{(-4-3)^2+(3-5)^2}$
$\sqrt{(-7)^2+(-2)^2}$
$\sqrt{49+4}$ $\sqrt{53}$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

|DC| (−4,3) (2,−2)
$\sqrt{(2+4)^2+(-2-3)^2}$
$\sqrt{(6)^2+(-5)^2}$
$\sqrt{36+25}$
$\sqrt{61}$
$= 7.81$

$\neq$

|BC| (8,2) (2,−2)
$\sqrt{(2-8)^2+(-2-2)^2}$
$\sqrt{(-6)^2+(-4)^2}$
$\sqrt{36+16}$
$\sqrt{52}$
$= 7.21$

6. Find the distance between each of the following pairs of points:

(i) (2, 1) and (3, 4)
$\sqrt{(3-2)^2+(4-1)^2}$
$\sqrt{(1)^2+(3)^2}$
$\sqrt{1+9}$
$= \sqrt{10}$
3.2

(ii) (1, 5) and (2, 3)
$\sqrt{(2-1)^2+(3-5)^2}$
$\sqrt{(1)^2+(-2)^2}$
$\sqrt{1+4}$
$\sqrt{5}$

(iii) (−1, 4) and (2, 6)
$\sqrt{(2-(-1))^2+(6-4)^2}$
$\sqrt{(3)^2+(2)^2}$
$\sqrt{9+4}$
Ans: $\sqrt{13}$

(iv) (3, −2) and (−5, 3)
$\sqrt{(-5-3)^2+(3+2)^2}$

(v) (−6, −1) and (1, −3)
$\sqrt{(1+6)^2+(-3+1)^2}$

(vi) (4, −2) and (0, −5)
$\sqrt{(0-4)^2+(-5+2)^2}$

(iv) $(3, -2)$ and $(-5, 3)$

$\sqrt{(-5-3)^2+(3+2)^2}$

$\sqrt{(-8)^2+(5)^2}$

$\sqrt{64+25} = \sqrt{89}$

(v) $(-6, -1)$ and $(1, -3)$

$\sqrt{(1+6)^2+(-3+1)^2}$

$\sqrt{(7)^2+(-2)^2}$

$\sqrt{49+4}$

$\sqrt{53}$

(vi) $(4, -2)$ and $(0, -5)$

$\sqrt{(0-4)^2+(-5+2)^2}$

$\sqrt{(-4)^2+(-3)^2}$

$\sqrt{16+9}$

$\sqrt{25} = 5$

7. A(1, 1), B(3, 6) and C(5, 1) are the vertices of a triangle. Show that |AB| = |BC|.

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

|AB|

A $(1,1)$ B $(3,6)$

$\sqrt{(3-1)^2+(6-1)^2}$

$\sqrt{(2)^2+(5)^2}$

$\sqrt{4+25}$

$\sqrt{29}$

|BC|

B $(3,6)$ C $(5,1)$

$\sqrt{(5-3)^2+(1-6)^2}$

$\sqrt{(2)^2+(-5)^2}$

$\sqrt{4+25}$

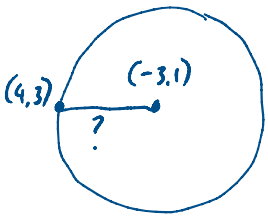
$\sqrt{29}$

$|AB| = |BC|$

$\sqrt{29} = \sqrt{29}$

8. The centre of a circle is $(-3, 1)$ and $(4, 3)$ is a point on the circle. Find the length of the radius of the circle.

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



$\sqrt{(4-(-3))^2+(3-1)^2}$

$+3$

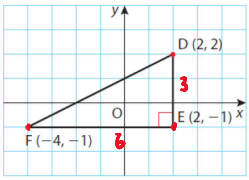
$\sqrt{(7)^2+(2)^2}$

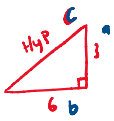
$\sqrt{49+4}$

$\sqrt{53}$

9. The points A(2, 1), B(6, 1), C(5, −2) and D(1, −2) are the vertices of a parallelogram.
Plot the parallelogram on a coordinated plane.
Find (i) |AC| (ii) |BD|.
Are the diagonals equal in length?

10. The given diagram shows the points D, E and F.





$c^2 = a^2 + b^2$
$c^2 = 3^2 + 6^2$
$c^2 = 9 + 36$
$c^2 = 45$
$c = \sqrt{45}$
$\sqrt{9 \times 5}$
$6.70 = 3\sqrt{5}$

6 3

(i) Write down the lengths of [FE] and [ED].
(ii) Find |DF|.

Use the Theorem of Pythagoras to show that the triangle DEF is right-angled.

Mid point

Log tables pg 18 $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

① Label the points (x_1, y_1) and (x_2, y_2)
② Sub values into the formula
③ Answer in a point (x, y)

11. Find the midpoint of the line segment joining these points:

(i) $(2, 4)$ and $(6, 2)$ — $x_1\ y_1$ and $x_2\ y_2$

Formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Sub into formula $\left(\frac{2+6}{2}, \frac{4+2}{2}\right)$ Tidy up

$\left(\frac{8}{2}, \frac{6}{2}\right)$

$(4, 3)$ midpoint

(ii) $(2, 4)$ and $(0, 2)$ — $x_1\ y_1$ and $x_2\ y_2$

$\left(\frac{2+0}{2}, \frac{4+2}{2}\right)$

$\left(\frac{2}{2}, \frac{6}{2}\right)$

$(1, 3)$

(iii) $(2, -1)$ and $(4, 3)$ — $x_1\ y_1$ and $x_2\ y_2$

$\left(\frac{2+4}{2}, \frac{-1+3}{2}\right)$

$\left(\frac{6}{2}, \frac{2}{2}\right) = (3, 1)$

(iv) $(-2, 4)$ and $(4, -2)$ — $x_1\ y_1$ and $x_2\ y_2$

$\left(\frac{-2+4}{2}, \frac{4-2}{2}\right)$

$\left(\frac{2}{2}, \frac{2}{2}\right)$

$= (1, 1)$

(v) $(2, -3)$ and $(0, -1)$ — $x_1\ y_1$ and $x_2\ y_2$

$\left(\frac{2+0}{2}, \frac{-3-1}{2}\right)$

$\left(\frac{2}{2}, \frac{-4}{2}\right)$

$= (1, -2)$

(vi) $(-3, 4)$ and $(-1, -4)$. — $x_1\ y_1$ and $x_2\ y_2$

$\left(\frac{-3-1}{2}, \frac{4-4}{2}\right)$

$\left(\frac{-4}{2}, \frac{0}{2}\right)$

$= (-2, 0)$

12. Find the midpoint of the line segment joining $(-3, 4)$ and $(3, 7)$. — $x_1\ y_1$ and $x_2\ y_2$
On which axis does the midpoint lie?

H / W

Mid point

Log tables Pg 18 Coordinate Geometry. (Point) (x, y)

Formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ Mid point – middle

Tue 17-9-19
C/W pg 206
Q11 (ii) → (vi)

Method for midpoint
1) Write out the formula
2) Label the points (x_1, y_1) (x_2, y_2)
3) Sub values into the formula.

Eg: Find the midpoint of the line segment joining the points $(2, 4)$ and $(6, 2)$ — $x_1\ y_1$ and $x_2\ y_2$

Mid point $\left(\frac{2+6}{2}, \frac{4+2}{2}\right) = \left(\frac{8}{2}, \frac{6}{2}\right) = (4, 3)$

13. The points $(-2, 3)$ and $(6, 5)$ are the end points of the diameter of a circle.
Find the coordinates of the centre of the circle.

H/W

14. A(4, 3), B(1, −3), C(−2, −2) and D(1, 4) are the vertices of a parallelogram.
Draw a sketch of this parallelogram.
Find the midpoint of [AC].
Verify that the midpoint of [AC] is also the midpoint of [BD].

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \Rightarrow \left(\frac{4-2}{2}, \frac{3-2}{2}\right)$

$\left(\frac{2}{2}, \frac{1}{2}\right)$

$(1, .5)$

BD =

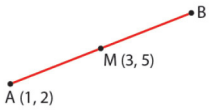
$\left(\frac{1+1}{2}, \frac{-3+4}{2}\right)$

$\left(\frac{2}{2}, \frac{1}{2}\right)$

$\Leftrightarrow (1, .5)$ ✓

15. Find M, the midpoint of the line segment joining A(−3, 4) and B(1, −6).
Now show that $|AM| = |MB|$.

16. The given diagram shows the points A(1, 2), M(3, 5) and B.
If M is the midpoint of [AB], find by inspection the coordinates of the point B.



17. A(5, 2), and B(x_1, y_1) are two points.
If M(2, 4) is the midpoint of [AB], find the coordinates of B.

Answers

Exercise 11.1

1. A(2, 3), B(4, 2), C(2, 1), D(−3, 3), E(−2, 2), F(−4, 1), G(−3, 0), H(−4, −2), I(−2, −3), J(0, −3), K(2, −3), L(3, −2), M(5, −3)

3. (i) First (ii) Third (iii) Fourth (iv) Second (v) Fourth (vi) Third

4. (i) *x*-axis (ii) *x*-axis (iii) *y*-axis (iv) *y*-axis (v) both

5. (i) $\sqrt{34}$ (ii) $\sqrt{50}$ (iii) $\sqrt{53}$; No

6. (i) $\sqrt{10}$ (ii) $\sqrt{5}$ (iii) $\sqrt{13}$ (iv) $\sqrt{89}$ (v) $\sqrt{53}$ (vi) 5

8. $\sqrt{53}$

9. (i) $\sqrt{18}$ (ii) $\sqrt{34}$; No

10. (i) $|FE| = 6$, $|ED| = 3$ (ii) $\sqrt{45}$

11. (i) (4, 3) (ii) (1, 3) (iii) (3, 1) (iv) (1, 1) (v) (1, −2) (vi) (−2, 0)

12. $(0, \frac{11}{2})$; *y*-axis

13. (2, 4)

14. $(1, \frac{1}{2})$

15. (−1, −1)

16. (5, 8)

17. (−1, 6)