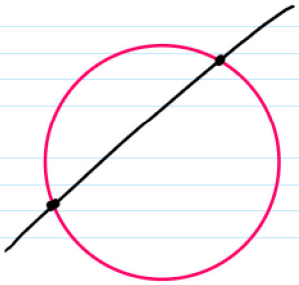
Intersection of a line and a circle

10 January 2020 16:53



The line cuts the circle at two points

Note: $2x$ ← variable, ↙ coefficient

Method: To find the two points of intersection of a line and a circle.

1) Rearrange the linear equation to get either x or y on its own (which ever has no coefficient)

2) Substitute the rearranged linear equation into the corresponding variable in the circle equation

3) Get rid of brackets and tidy up. Factorize the quatratic equation and solve to find missing values.

4) Sub the values found in (3) back into linear equation

Eg1) A circle $x^2+y^2=40$ and a line $x+y-4=0$

Find the points of intersection

① Linear Equation $x+y-4=0$

$+4$ | $x+y=4$ | $+4$

$-y$ | $x=-y+4$ | $-y$

In terms of x. x on its own

② Sub in $x=-y+4$ into x part of $x^2+y^2=40$

$(-y+4)^2+y^2=40$

③ $-y(-y+4)+4(-y+4)+y^2=40$ Tidy up and factorize.

$y^2-4y-4y+16+y^2=40$

-40 | $2y^2-8y+16-40=0$ | -40

$2y^2-8y-24=0$ Quadratic ÷2 to simplify

$y^2-4y-12=0$

$(y-6)(y+2)=0$ $+2y$ $-6y$

$(y-6)(y+2)=0$ $+2y$ $-6y$

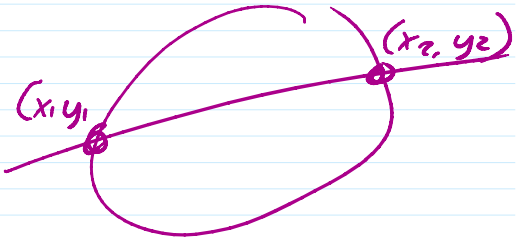
$-4y$

Solve for y

$y-6=0$ | $+6$ | $y=6$

$y+2=0$ | -2 | $y=-2$

(4) Sub the values of y back into the rearranged linear equation



$x=-y+4$

$y=6$

$x=-(6)+4$

$x=-2$

$(-2,6)$

$y=-2$

$x=-(-2)+4$

$x=+2+4$

$x=6$

$(6,-2)$