Inequalities

28 January 2019 09:12

Log Tables – Sets and Logic Pg 23

Number Systems Revision

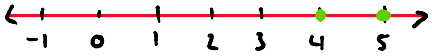
① Natural numbers $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

Number line

"use dots"

Eg) $x > 3$

$3 < x$



② Integers $\mathbb{Z} = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

positive and negative whole numbers

Number line

"use dots"

Eg2) Show the inequality $-3 \leq x \leq 1$, $x \in \mathbb{Z}$

③ The real numbers $\mathbb{R}$ → All numbers fractions decimals, whole numbers.

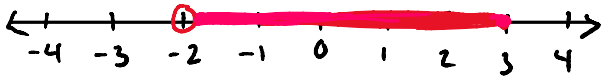
Number line

$x \geq -3$, $x \in \mathbb{R}$

R - River

Heavy thick line.

Eg 3) Show on a numberline $-2 < x \leq 3$, $x \in R$



Solving inequalities

Eg 1 Solve the inequality and graph on a number line.

$3x - 2 \leq 7$, $x \in N$

$+2$ | $3x \leq 9$ | $+2$

$\div 3$ | $x \leq 3$ | $\div 3$

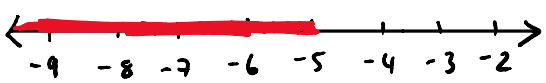
$x = \{1, 2, 3\}$

Eg2) $3 - 2(4 - x) \geq 3x$, $x \in \mathbb{R}$. Solve for x and graph on a number line

$3 - 8 + 2x \geq 3x$

$-5 + 2x \geq 3x$

$-2x$ | $-5 \geq 1x$ | $-2x$



Classwork Pg 13 Q4+5

Negative inequalities

Method ① Get rid of the minus on the x by multiplying across the inequality by -1

② The direction of the inequality sign changes.

Eg1) (-1) multiply $7 - x > 4$, $x \in \mathbb{Z}$

$-7 + x < -4$

$+7$ | $x < 3$ | $+7$

T&T3 1.7

T&T3 1.7.pptx

PROJECT MATHS

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Leaving 3 Certificate

Section 1.7 Linear inequalities

Notes

$2x + 4 = 6$ is an example of an equation because both sides are equal.
However $2x + 4 > 6$ is an **inequality** as both sides are **not** equal.

The four inequality symbols are as follows

1. $>$ … is greater than
2. $\geqslant$ is greater than or equal to
3. $<$ … is less than
4. $\leqslant$ is less than or equal to

Notes

The rules for solving inequalities are very similar to those for solving equations. There is, however, one important difference.

The inequality sign is reversed when both sides are multiplied or divided by the same **negative** number.

$3 < 5$ but $3 \times (-1) > 5 \times (-1)$

i.e. $-3 > -5$

$(-3 < -5)$ not true.

Negative inequalities

When you have a minus x part, you need to make this positive

Multiply across All of the inequality by -1

The direction of the inequality sign changes direction

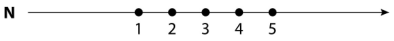
$< \times (-1) = >$

$> \times (-1) = <$

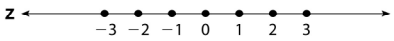
Notes

Before solving inequalities, we will revisit the different types of number and how they are represented on the number line.

1. The set of natural numbers, $\mathbf{N} = \{1, 2, 3, 4, \ldots\}$

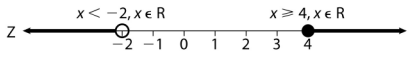


2. The set of integers, $\mathbf{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$



3. The set of real numbers **R** contains all the numbers on the number line. R is represented on the number line by a **bold** line which indicates that all the numbers are included.

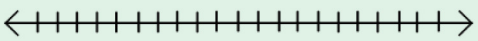
Here are two sets of real numbers represented on the number line.



The full circle at 4 indicates that 4 is included.
The empty circle at -2 indicates that -2 is not included.

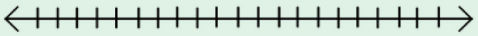
Example 1

Solve the inequality $5 - 2x < 9, x \in Z$ and illustrate the solution on the number line.



Example 2

Find the solution set A of $3x + 1 \leqslant 2x + 5,\ x < R$.
Find the solution set B of $\frac{1}{3} - 2x \leqslant \frac{25}{3},\ x < R$.
Find $A \cap B$ and illustrate your answer on the number line.



Exercise 1.7

Answer: A, F; B, D; C, H; E, G

1. Match up these inequalities.

A $2x < 10$

B $x > 4$

C $\frac{x}{2} > 3$

D $x + 3 > 7$

E $x + 1 < 5$

F $x < 5$

G $x + 3 < 7$

H $x > 6$

Exercise 1.7

Answer: D and E

2. Which two of the following are equivalent to $m > 6$?

A $2m > 8$

B $\frac{m}{2} > 12$

C $3m > 2$

D $2m > 12$

E $\frac{m}{3} > 2$

F $\frac{m}{2} > 4$

Exercise 1.7 **Answer:** A, D; B, G; F, H; C, E

3. Find the four equivalent pairs in these eight inequalities.

A $p \leqslant 8$ B $4p \leqslant 24$ C $2p \leqslant 18$ D $p - 2 \leqslant 6$ E $p \leqslant 9$

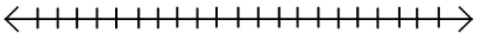
F $p + 4 \leqslant 14$ G $3p \leqslant 18$ H $2p \leqslant 20$

13

Exercise 1.7 **Answer:** $x \leqslant 3$

Solve the following inequalities and graph the solution set on the number line:

4. $3x - 2 \leqslant 7, \quad x \in N$



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Exercise 1.7 **Answer:** $x < -3$

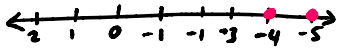
Solve the following inequalities and graph the solution set on the number line:

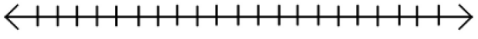
5. $8x - 1 < 5x - 10, \quad x \in Z$

$-5x$ | $3x - 1 < -10$ | $-5x$

$+1$ | $3x < -9$ | $+1$

$\div 3$ | $x < -3$ | $\div 3$





13

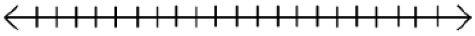
Exercise 1.7 — **Answer:** $x \leq 4$

Solve the following inequalities and graph the solution set on the number line:

6. $3x + 1 \leq 2x + 5, \quad x \in Z$ Dots

$-2x \mid 1x + 1 \leq 5 \mid -2x$

$-1 \mid x \leq 4 \mid -1$



Exercise 1.7 — **Answer:** $x < 3$

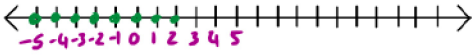
Solve the following inequalities and graph the solution set on the number line:

7. $7 - x > 4, \quad x \in Z$ (-1) multiply

$> \Rightarrow <$ change direction

$-7 + x < -4$

$+7 \mid x < 3 \mid +7$



Exercise 1.7 — **Answer:** $x \leq -3$

Solve the following inequalities and graph the solution set on the number line:

8. $2x - 5 \geq 3x - 2, \quad x \in R$

$-2x \mid -5 \geq x - 2 \mid -2x$

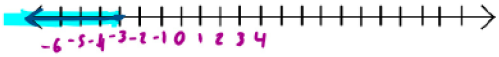
$+2 \mid -3 \geq x \mid +2$

$-3x \mid -x - 5 \geq -2 \mid -3x$

$+5 \mid -x \geq +3 \mid +5$

$(-1) \mid x \leq -3 \mid (-1)$

R- river



Exercise 1.7 — **Answer:** $x < -12$

Solve the following inequalities and graph the solution set on the number line:

9. $4(x + 2) < 3x - 4, \quad x \in R$

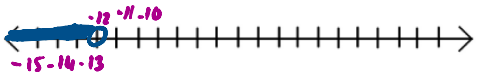
$4x + 8 < 3x - 4$

$-3x \quad -3x$

$-8 \quad -8$

$1x + 8 < -4$

$x < -12$



Exercise 1.7 — **Answer:** $x \geq 4$

Solve the following inequalities and graph the solution set on the number line:

10. $1 \leq 3x - 11, \quad x \in Z$

$+11 \quad +11$

$\div 3 \quad \div 3$

$12 \leq 3x$

$4 \leq x$

Exercise 1.7 — **Answer:** $x \leq 1$

Solve the following inequalities and graph the solution set on the number line:

11. $7 - 4x \geq 2x + 1, \quad x \in R$

$-1 \quad -1 \quad -x$

$+4x \quad +4x \quad +4x$

$\div 6 \quad \div 6$

$6 - 4x \geq 2x$

$6 \geq 6x$

$1 \geq x$

Exercise 1.7 **Answer:** $x \leqslant -5$

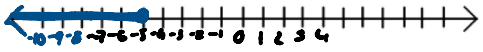
Solve the following inequalities and graph the solution set on the number line:

12. $3 - 2(4 - x) \geqslant 3x, \quad x \in R$

$3 - 8 + 2x \geq 3x$

$-5 + 2x \geq 3x$ $\quad -2x \quad -2x$

$-5 \geq x$



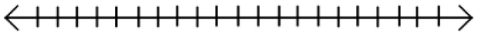
HW

Pg 13 Q13-16

Exercise 1.7 **Answer:** $x \leqslant -8$

Solve the following inequalities and graph the solution set on the number line:

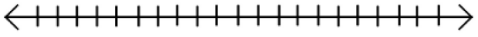
13. $3x \leqslant -2(4 - x), \quad x \in R$



Exercise 1.7 **Answer:** $x \geqslant -1$

Solve the following inequalities and graph the solution set on the number line:

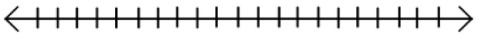
14. $x - 4 \leqslant 4x - 1, \quad x \in R$



Exercise 1.7 **Answer:** $x > 2$

Solve the following inequalities and graph the solution set on the number line:

15. $4 - 2x > 5(2 - x), \quad x \in R$



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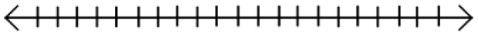
Exercise 1.7 **Answer:** $x \geqslant 1$

16. Solve the inequality $5(2x - 5) \geqslant 1 - 2(11 - 3x), \quad x \in R.$
Now illustrate your solution on the number line.



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Exercise 1.7 **Answer:** $-3 < x \leqslant 3$

17. Find the solution set K of $11 \geqslant 3x + 2, \quad x \in R.$
Find the solution set L of $3x + 2 > -7, \quad x \in R.$
Find $K \cap L$ and graph the solution on the number line.

K
$11 \geq 3x+2$
$-2 \mid 9 \geq 3x \mid -2$
$\div 3 \mid 3 \geq x \mid \div 3$

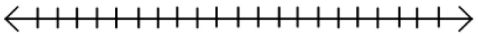
L
$3x+2 > -7$
$-2 \mid 3x > -9 \mid -2$
$\div 3 \mid x > -3 \mid \div 3$

$-3 < x \leq 3$
$\{-2,-1,0,1,2,3\}$

13

Exercise 1.7 **Answer:** $-\frac{1}{2} \leqslant x \leqslant 3$

18. Find the solution set P of $2 - 3x \leqslant 4 + x, \quad x \in R$.
Find the solution set Q of $4 + x \leqslant 7, \quad x \in R$.
Find $P \cap Q$ and graph the solution on the number line.



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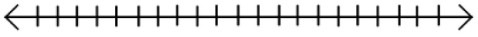
Exercise 1.7 **Answer:** $\frac{1}{2} < x \leqslant 8$

19. Find the solution set A of $x < 3x - 1, \quad x \in R$.
Find the solution set B of $3x - 1 \leqslant 2x + 7, \quad x \in R$.
Now illustrate $A \cap B$ on the number line.



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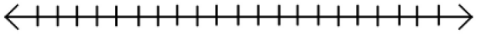
Exercise 1.7 **Answer:** $x = 2, 3, 4$

20. Find the solution set C of $2 \leqslant \frac{5x - 6}{2}, \quad x \in N$.

Find the solution set D of $\frac{5x - 6}{2} \leqslant 7, \quad x \in N$.

Illustrate $C \cap D$ on the number line.



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Exercise 1.7 — **Answers:** (i) (b) is true (ii) 9

21. A bus can carry a maximum of 44 passengers.
A school wants to take 5 adults and as many groups of 4 children as possible on the bus.

(i) Given that n represents the number of children, which of these inequalities is true for the bus

(a) $4n + 5 \geqslant 44$ (b) $4n - 5 \leqslant 44$

(c) $4n - 5 < 44$ (d) $4n + 5 > 44$?

(ii) Solve the inequality to find the maximum number of groups of 4 children the bus can carry.

13

Q6 2019 Paper 1

$2(3-x) < 8 \quad x \in \underline{R}$

$6 - 2x < 8$

-6 | $-2x < 2$ | -6
(-1) | $2x > -2$ | (-1)
$\div 2$ | $x > -1$ | $\div 2$

c) $2(2x-3)+6x < 25 \quad x \in N$

$4x - 6 + 6x < 25$

$10x - 6 < 25$

$+6$ | $10x < 31$ | $+6$
$\div 10$ | $x < 3.1$ | $\div 10$

$x = \{1, 2, 3\}$

b) $2 - 3x \geq -6 \quad x \in \underline{N}$

-2 | $-3x \geq -8$ | -2
(-1) | $3x \leq 8$ | (-1)
$\div 3$ | $x \leq 2.6$ | $\div 2$

$x = \{1, 2\}$