10. Indices

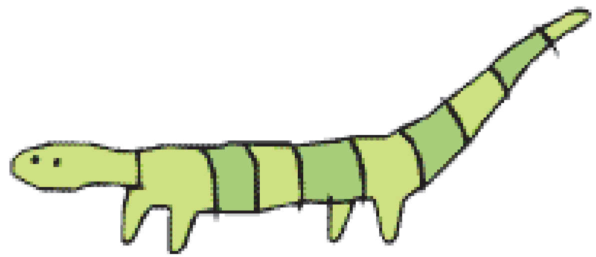
What are Indices?

Indices are just a fancy word for "power"

They are the little numbers or letters that float happily in the air next to a number or letter

A bit of indices lingo:

The base → 5^4 ← The index or power



Two things you must remember about indices...

1. Indices only apply to the number or letter they are to the right of – the base

e.g. in abc^2, the squared only applies to the c, and nothing else. If you wanted the squared to apply to each term, it would need to be written as $(abc)^2$.

2. Indices definitely do not mean multiply

e.g. 6^3 definitely does not mean 6×3, it means $6 \times 6 \times 6$!

Return to contents page

Rule 1 – The Multiplication Rule

Using fancy notation: $a^m \times a^n = a^{m+n}$

What it actually means: Whenever you are multiplying two terms with the same base, you can just add the powers!

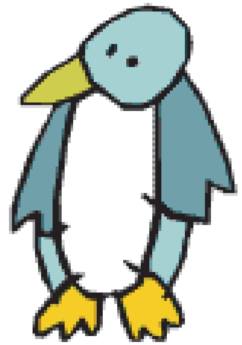
Numbers: If there are numbers IN FRONT of your bases, then you must multiply those numbers together as normal

Examples

$x^3 \times x^4 = x^7$ √ Classic wrong answer: x^{12} ×

$2^5 \times 2^3 = 2^8$ √ Classic wrong answer: 4^8 ×

$3p^4 \times 2p^5 = 6p^9$ √ Classic wrong answer: $6p^{20}$ ×



$2ab^2c \times 5ab^2c^3 = 10a^2b^4c^4$ √

Remember: if a base does not appear to have a power, the power is a disguised 1!

e.g. $2ab^2c = 2a^1b^2c^1$

Return to contents page

Rule 2 – The Division Rule

Using fancy notation: $a^m \div a^n = a^{m-n}$ Or $\frac{a^m}{a^n} = a^{m-n}$

What it actually means: Whenever you are dividing two terms with the same base, you can just subtract the powers!

Numbers: If there are numbers IN FRONT of your bases, then you must divide those numbers as normal

Examples

$x^{12} \div x^4 = x^8$ √ Classic wrong answer: x^3 ×

$\frac{5^7}{5^3} = 5^4$ √ Classic wrong answer: 1^4 ×

$\frac{20k^{10}}{5k^5} = 4k^5$ √ Classic wrong answer: $4k^2$ ×

Return to contents page

Rule 3 – The Power of a Power Rule

Using fancy notation: $(a^m)^n = a^{m \times n}$

What it actually means: Whenever you have a base and it's power raised to another power, you simply multiply the powers together but keep the base the same!

Numbers: If there is a number IN FRONT of your base, then you must raise that number to the power

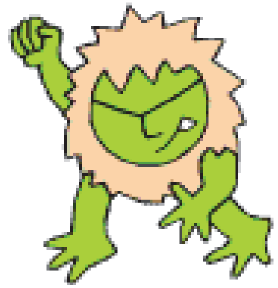
Examples

$(x^5)^3 = x^{15}$ √ Classic wrong answer: x^8 ✗

$(2^3)^2 = 2^6$ √ Classic wrong answer: 4^6 ✗

$(3a^4)^3 = 27a^{12}$ √ Classic wrong answer: $9a^{12}$ ✗

$(2a^3b^2c)^5 = 32a^{15}b^{10}c^5$ √



Return to contents page

Examples Using all Three Rules

Rule 1: $a^m \times a^n = a^{m+n}$

Rule 2: $\dfrac{a^m}{a^n} = a^{m-n}$

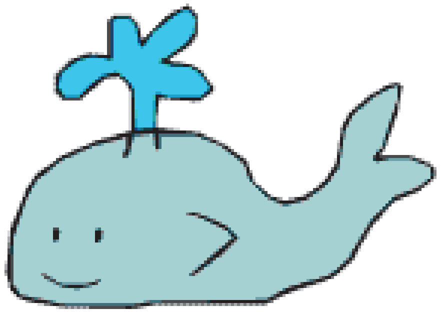
Rule 3: $(a^m)^n = a^{m \times n}$

1. $\dfrac{x^3 \times (x^2)^4}{x^5}$ —Rule 3→ $\dfrac{x^3 \times x^8}{x^5}$ —Rule 1→ $\dfrac{x^{11}}{x^5}$ —Rule 2→ x^6

2. $\dfrac{(5^3)^2 \times (5^2)^{10}}{(5^5)^2 \times 5}$ —Rule 3→ $\dfrac{5^6 \times 5^{20}}{5^{10} \times 5^1}$ —Rule 1→ $\dfrac{5^{26}}{5^{11}}$ —Rule 2→ x^{15}

3. $\dfrac{(5v^4)^2 \times (2v^5)^4}{50v}$ —Rule 3→ $\dfrac{25v^8 \times 16v^{20}}{50v}$ —Rule 1→ $\dfrac{400v^{28}}{50v^1}$

—Rule 2→ $8v^{27}$



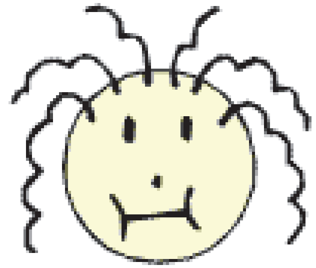
Return to contents page

Rule 4 – The Zero Index

Using fancy notation: $a^0 = 1$

What it actually means: Anything to the power of zero is 1!

Examples $x^0 = 1$ $\quad 17^0 = 1$ $\quad 5x^0 = 5 \times 1 = 5$



Rule 5 – Negative Indices

Using fancy notation: $a^{-m} = \frac{1}{a^m}$

What it actually means: A negative sign in front of a power is the same as writing "one divided by the base and power". The posh name for this is the RECIPROCAL

Watch out! Only the power and base are flipped over, nothing else!

Examples $x^{-2} = \frac{1}{x^2}$ $\quad 5^{-4} = \frac{1}{5^4}$ $\quad 5a^{-3} = \frac{5}{a^3}$

$$\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3 \qquad \left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 16 \qquad \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

Return to contents page

Rule 6 – Fractional Indices

Using fancy notation: $a^{\frac{1}{n}} = \sqrt[n]{a}$

What it actually means: When a power is a fraction it means you take the root of the base... and which root you take depends on the number on the bottom of the fraction!

The main ones:

$a^{\frac{1}{2}} = \sqrt{a}$ The power of a half means take the square-root!

$a^{\frac{1}{3}} = \sqrt[3]{a}$ The power of a third means take the cube-root!

Examples

$64^{\frac{1}{2}} = \sqrt{64} = 8$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ Because $3^3 = 27$

$32^{\frac{1}{5}} = \sqrt[5]{32} = 2$ Because $2^5 = 32$

For ones like the last two it is worth learning your powers of 2 and 3:

$2^2 = 4$ $3^2 = 9$

$2^3 = 8$ $3^3 = 27$

$2^4 = 16$ $3^4 = 81$

$2^5 = 32$

$2^6 = 64$

Return to contents page

Flip It, Root It, Power It!

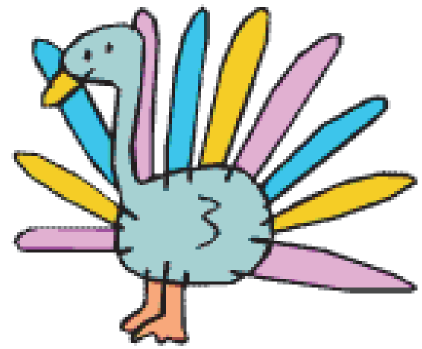
Sometimes you get asked some indices questions that look an absolute nightmare, but if you just deal with each aspect in turn, then you will be fine:

1. Flip It – If there is a negative sign in front of your power, flip the base over and we're positive!
2. Root It – If your power is a fraction, then deal with the bottom of it by rooting your base
3. Power It – When all that is sorted, just raise your base to the remaining power and you're done!

Examples

1. $8^{-\frac{2}{3}} \xrightarrow{\text{Flip it}} \left(\frac{1}{8}\right)^{\frac{2}{3}} \xrightarrow{\text{Root It}} \left(\frac{\sqrt[3]{1}}{\sqrt[3]{8}}\right)^2 = \left(\frac{1}{2}\right)^2$

   $\xrightarrow{\text{Power It}} \frac{1^2}{2^2} = \frac{1}{4}$



2. $\left(\frac{1}{64}\right)^{-\frac{5}{6}} \xrightarrow{\text{Flip it}} 64^{\frac{5}{6}} \xrightarrow{\text{Root It}} (\sqrt[6]{64})^5 = 2^5$

   $\xrightarrow{\text{Power It}} 32$

Return to contents page