Indices Intro

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Indices, Surds and Logarithms

Indices

Laws

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \left(\frac{a^n}{b^n}\right)$

$a^{-m} = \frac{1}{a^m}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

Special Results

$a^0 = 1$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

$*(a \pm b)^2 \neq a^2 \pm b^2$

Logarithms

Definition

$\log_a x = y \Rightarrow x = a^y$

Laws

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

$\log_a x^r = r \log_a x$

$\log_a b = \frac{\log_c a}{\log_c b}$

Special Results

$\ln = \log_e$ $*(e \approx 2.71...)$

$\lg = \log_{10}$

$\log_a a = 1$

$\log_a 1 = 0$

$\log_a x, \ x > 0$

Surds

Simplification

a) $\frac{1}{\sqrt{2}}$

$= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$= \sqrt{2}$

b) $\sqrt{12}$

$= \sqrt{3 \times 4}$

$= \sqrt{3} \times \sqrt{4}$

$= 2\sqrt{3}$

c) $\frac{1}{2-\sqrt{3}}$

$= \frac{1 \times (2+\sqrt{3})}{(2-\sqrt{3}) \times (2+\sqrt{3})}$

$= \frac{2+\sqrt{3}}{2^2 - (\sqrt{3})^2}$

$= \frac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$

Log Tables
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1) What are indices?

Powers → how many time a number / letters is multipled by itself

2) Indices can also be called what?

Powers, Exponents, Index form.

3) In the order of operations where does indices come in?

B $\underbrace{\text{I R}}$ D M A S

Indices ↔ Roots

- Answers :

1) An indice : is a letter or a prime number written to a power

We call the letter or the prime number the **BASE VALUE**

Eg 1) a^3 ← Power

↓ Base

Eg 2) x^2 ← Power

↓ Base Value

Base
Value

Base
Value

Eg 3) x

Is there a power on this base value?

yes → to the power of 1

When no power is written the value of the power is 1

(x^1) by $(x^1) = x^2$

Rule 1) To multiply powers of the same base value you add the powers and leave the base value the same

Log Table Rule : $a^p \times a^q = a^{p+q}$

a is the base value

p and q are the powers.

Eg 1) $5^3 \times 5^6 = 5^{3+6} = 5^9$

Eg 2) $(2^3)(2^5) = 2^{3+5} = 2^8$

Brackets mean - multiply

Add powers

Eg 3) $3^2 . 3^4 = 3^{2+4} = 3^6$

A dot can also mean multiply

mean
multiply

Q1) Write each in the form a^n, where
$n \in \mathbb{N}$

1) $a^9 \times a^2 =$

2) $a \times a^3 =$

3) $(a^4)(a^7) =$

Rule 2: To divide powers of the same base value, SUBTRACT the powers. Base value stays the same

Log Table Rule : $\frac{a^p}{a^q} = a^{p-q}$ subtract

You subtract the bottom power from the top

Eg 1) $\frac{a^p}{a^q}$ subtract $= a^{p-q}$

Eg 2) $\frac{2^7}{2^4}$ Subtract the powers $= 2^{7-4} = 2^3$

subtract
3^4 $4-1=3$ 3

Eg 3) $3^4 \div 3 = ?$ $\frac{3^4}{3^1}$ } subtract powers $4-1=3$ 3^3

Notes into hard back.

Log tables pg 21

Rule 3: $(a^p)^q = a^{pq}$ A bracket to a power

Multiply the powers

Eg 1) $(a^2)^3 = a^{2\times 3} = a^6$ multiply powers

Eg 2) $(a^3)^4 = a^{3\times 4} = a^{12}$

When a number and a letter are involved we apply the power to both parts seperately.

Eg 3) $(3a)^2 \Rightarrow 3^2 \times a^2 = 9a^2$

Rule 4) The power of 0 (ZERO)

$a^0 = 1$

Note : Any number or letter to the power of zero is ALWAYS 1

Eg 1) $\frac{a^3}{}$ } subtract powers when dividing

Eg 1) $\dfrac{a^3}{a^3}$ subtract powers when dividing

$$a^{3-3} = a^0 = 1$$

Rule 5: $a^{-p} = \dfrac{1}{a^p}$ negative power

Eg 1) $$\frac{a^2}{a^5} = \frac{\cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times a \times a \times a} = \frac{1}{a^3}$$

OR using division law 2 : $\dfrac{a^2}{a^5}$ subtract powers

$$= a^{2-5} = a^{-3}$$

So $\dfrac{1}{a^3} = a^{-3}$

– Note if the power is negative – make a reciprocal by putting 1 over the indice and make the power positive.

Reciprocal $= \dfrac{1}{\text{number}}$

Law 6 : powers as roots.

$$a^{\frac{1}{q}} = \sqrt[q]{}$$

Note : $\frac{1}{2} = \sqrt{}$ square root, $\frac{1}{3} = \sqrt[3]{}$ cube root, $\frac{1}{4} = \sqrt[4]{}$ fourth root ...

Eg's) i) $4^{1/2} = \sqrt{4} = 2$ iii) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

ii) $8^{1/3} = \sqrt[3]{8} = 2$ iv) $32^{\frac{1}{5}} = \sqrt[5]{32} = 2$

The denominator will ALWAYS be the value infront of the root sign.

On your calculator keys [$\sqrt{\blacksquare}$] square root

[SHIFT] [$\sqrt{\blacksquare}$] = $\sqrt[3]{\blacksquare}$ cube root

Any other root [SHIFT] [$x^{\blacksquare}$] = $\sqrt[\blacksquare]{\square}$ use the arrow key to move around and enter values.

Law 7 : Fractional Power

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

The numerator (p) becomes the power and the denominator (q) becomes the root $\left(\sqrt[q]{a}\right)^p$

Eg 1) $4^{3/2}$ => $\frac{1}{2} = \sqrt{}$ square root

$= \left(\sqrt{4}\right)^3$

$= (2)^3 = 8$

Eg 2) $125^{2/3} = \frac{1}{3} = \sqrt[3]{}$

$\left(\sqrt[3]{125}\right)^2$

$(5)^2 = 25$

Law 8 : $(ab)^p = a^p b^p$ product is raised to a power

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Eg 1) $(xy)^3 \Rightarrow (x^3 y^3)$

Eg 2) $(2x)^3 \Rightarrow 8x^3$

$2^3 x^3$

Law 9: $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$ when a fraction is raised to a power.

Each part of the fraction (quotient) is raised to the power

Eg 1) $\left(\frac{4}{2}\right)^3 = \frac{4^3}{2^3} = \frac{64}{8} = 8$

Eg 2) $\left(\frac{2xy}{x}\right)^2 \Rightarrow \frac{2^2x^2y^2}{x^2} \Rightarrow \frac{4\cancel{x^2}y^2}{\cancel{x^2}} = 4y^2$

Eg 3) $\left(\frac{3x}{4y}\right)^3 \Rightarrow \frac{3^3x^3}{4^3y^3} = \frac{27x^3}{64y^3}$

HW Pg 327 Q 6.