

Indices, Surds and Logarithms

Indices <i>Laws</i>	Logarithms <i>Definition</i>	Surds <i>Simplification</i>
$a^n \times a^m = a^{n+m}$	$\log_a x = y \Rightarrow x = a^y$	a) $\frac{1}{\sqrt{2}}$
$\frac{a^n}{a^m} = a^{n-m}$	<i>Laws</i>	$= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$
$(a^n)^m = a^{nm}$	$\log_a x + \log_a y = \log_a xy$	$= \frac{1}{\sqrt{2}}$
$(ab)^n = a^n b^n$	$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$	b) $\sqrt{12}$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\log_a x^r = r \log_a x$	$= \sqrt{3 \times 4}$
$a^{-n} = \frac{1}{a^n}$	$\log_a b = \frac{\log_x a}{\log_x b}$	$= \sqrt{3} \times \sqrt{4}$
$a^{\frac{1}{n}} = \sqrt[n]{a} = (\sqrt[n]{a})^1$	<i>Special Results</i>	$= 2\sqrt{3}$
<i>Special Results</i>	$\ln = \log_e$ ($e = 2.718...$)	c) $\frac{1}{2-\sqrt{3}}$
$a^2 = 1$	$\lg = \log_{10}$	$= \frac{1 \times (2 + \sqrt{3})}{(2 - \sqrt{3}) \times (2 + \sqrt{3})}$
$(a \pm b)^2 = a^2 \pm 2ab + b^2$	$\log_a a = 1$	$= \frac{2 + \sqrt{3}}{2^2 - (\sqrt{3})^2}$
$(a+b)(a-b) = a^2 - b^2$	$\log_a 1 = 0$	$= \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$
$(a \pm b)^2 = a^2 \pm b^2$	$\log_a x, x > 0$	

Log Tables

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1) What are indices?

Powers \rightarrow how many times a number / letters is multiplied by itself

2) Indices can also be called what?

Powers, Exponents, Index form.

3) In the order of operations where does indices come in?

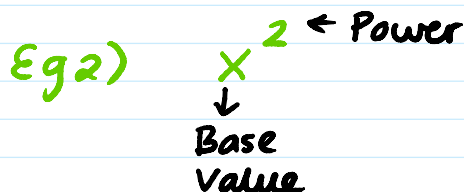
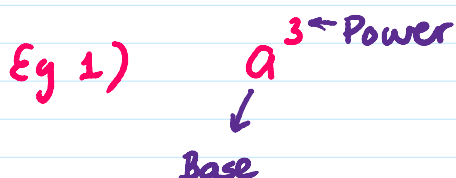
B **I** **R** **D** **M** **A** **S**

Indices \leftrightarrow Roots

Answers :

1) An indice : is a letter or a prime number written to a power

We call the letter or the prime number the BASE VALUE



↓
Base
Value

↓
Base
Value

Eg 3) x

Is there a power on this base value?

Yes → to the power of 1

When no power is written the value of the power is 1

$$(x^1) \text{ by } (x^1) = x^2$$

Rule 1) To multiply powers of the same base value you add the powers and leave the base value the same

Log Rule : $a^p \times a^q = a^{p+q}$
Table

a is the base value

p and q are the powers.

Eg 1) $5^3 \times 5^6 = 5^{3+6} = 5^9$

Eg 2) $(2^3)(2^5) = 2^{3+5} = 2^8$

Brackets mean - multiply Add powers

Eg 3) $3^2 \cdot 3^4 = 3^{2+4} = 3^6$

↓
A dot
can also
mean
multiply

mean
multiply

Q1) Write each in the form a^n , where
 $n \in \mathbb{N}$

$$1) a^9 \times a^2 =$$

$$2) a \times a^3 =$$

$$3) (a^4)(a^7) =$$

Rule 2: To divide powers of the same base value,
SUBTRACT the powers. Base value stays the same

Log Rule : $\frac{a^p}{a^q} \overset{\text{subtract}}{=} a^{p-q}$

You subtract the bottom power from
the top

$$\text{Eg 1) } \frac{a^p}{a^q} \overset{\text{subtract}}{=} a^{p-q}$$

$$\text{Eg 2) } \frac{2^7}{2^4} \overset{\text{subtract the powers}}{=} 2^{7-4} = 2^3$$

$$3^4 \overset{\text{subtract}}{=} 4-1=3 \quad 3$$

$$\text{Eg 3)} \quad 3^4 \div 3 = ? \quad \frac{3^4}{3^1} \quad \left. \begin{array}{l} \text{subtract} \\ 4-1=3 \\ \text{powers} \end{array} \right\} 3^3$$

Notes into hard back.

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Rule 3: $(a^p)^q = a^{pq}$ A bracket to a power
Multiply the powers

$$\text{Eg 1)} \quad (a^2)^3 = a^{2 \times 3} = a^6 \quad \text{multiply powers}$$

$$\text{Eg 2)} \quad (a^3)^4 = a^{3 \times 4} = a^{12}$$

When a number and a letter are involved we apply the power to both parts separately.

$$\text{Eg 3)} \quad (3a)^2 \Rightarrow 3^2 \times a^2 = 9a^2$$

—

Rule 4) The power of 0 (ZERO)

$$a^0 = 1$$

Note: Any number or letter to the power of zero is **ALWAYS 1**

$$\text{Eg 1)} \quad \frac{a^3}{a^1} \quad \left. \right\} \text{subtract powers when dividing}$$

Eg 1) $\frac{a^3}{a^3}$ } subtract powers when dividing

$$a^{3-3} = a^0 = 1$$

Rule 5: $a^{-p} = \frac{1}{a^p}$ negative power

Eg 1) $\frac{a^2}{a^5} = \frac{\cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times a \times a \times a} = \frac{1}{a^3}$

OR using division law 2 : $\frac{a^2}{a^5}$ } subtract powers
 $= a^{2-5} = a^{-3}$

so $\frac{1}{a^3} = a^{-3}$

Note if the power is negative - make a reciprocal by putting 1 over the indice and make the power positive.

$$\text{Reciprocal} = \frac{1}{\text{number}}$$

Law 6: powers as roots.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Note: $\frac{1}{2} = \sqrt{\quad}$, $\frac{1}{3} = \sqrt[3]{\quad}$, $\frac{1}{4} = \sqrt[4]{\quad}$...
square root cube root fourth root

Eg's) i) $4^{1/2} = \sqrt{4} = 2$

iii) $16^{1/4} = \sqrt[4]{16} = 2$

ii) $8^{1/3} = \sqrt[3]{8} = 2$

iv) $32^{1/5} = \sqrt[5]{32} = 2$

The denominator will ALWAYS be the value in front of the root sign.

On your calculator keys $[\sqrt{\square}]$ square root

$[\text{SHIFT}] [\sqrt{\square}] = \sqrt[3]{\square}$ cube root

Any other root

$[\text{SHIFT}] [x^{\square}] = \square\sqrt{\square}$ use the arrow key to move around and enter values.

Law 7: Fractional Power

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$$

The numerator (p) becomes the power and the denominator (q) becomes the root $(\sqrt[q]{a})^p$

Eg 1) $4^{3/2} \Rightarrow \frac{1}{2} = \sqrt{\quad}$ square root
 $= (\sqrt{4})^3$
 $= (2)^3 = 8$

Eg 2) $125^{2/3} = \frac{1}{3} = \sqrt[3]{\quad}$
 $(\sqrt[3]{125})^2$
 $(5)^2 = 25$

Law 8 : $(ab)^p = a^p b^p$ product is raised to a power

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Eg 1) $(xy)^3 \Rightarrow (x^3 y^3)$

Eg 2) $(2x)^3 \Rightarrow 2^3 x^3 = 8x^3$

Law 9: $\left(\frac{a}{b}\right)^P = \frac{a^P}{b^P}$ when a fraction is raised to a power.

Each part of the fraction (quotient) is raised to the power

Eg 1) $\left(\frac{4}{2}\right)^3 = \frac{4^3}{2^3} = \frac{64}{8} = 8$

Eg 2) $\left(\frac{2xy}{x}\right)^2 \Rightarrow \frac{2^2 x^2 y^2}{x^2} \Rightarrow \frac{4 \cancel{x^2} y^2}{\cancel{x^2}} = 4y^2$

Eg 3) $\left(\frac{3x}{4y}\right)^3 \Rightarrow \frac{3^3 x^3}{4^3 y^3} = \frac{27x^3}{64y^3}$

HLW Pg 327 Q 6.