

## Functions

A function is just an input output machine.

Patterns  $n^{\text{th}}$  term rule.

$$T_n = 2n + 1$$

$$T_1 = 2(1) + 1 = 3 \quad \downarrow 2$$

$$T_2 = 2(2) + 1 = 5 \quad \downarrow 2$$

$$T_3 = 2(3) + 1 = 7 \quad \downarrow 2$$

common  
diff

linear  
patter

Function will be  
written as

$$f(x) = 2x + 1$$

$$y = 2x + 1$$

function notation  $f(x)$

$y$

$x$  value is the input and  $y$  will be the output

↓ input    ↓ output.

$$f(x) = 2x + 1$$

$$f(1) = 3$$

when  $x$  is 1  $y$  is 3.

$$f(1) = 2(1) + 1$$

$$= 2 + 1$$

$$= 3$$

3, 5, 7  
↑    ↑  
+2 +2

slope.

$$f(2) = 2(2) + 1$$

$$4 + 1$$

$$= 5$$

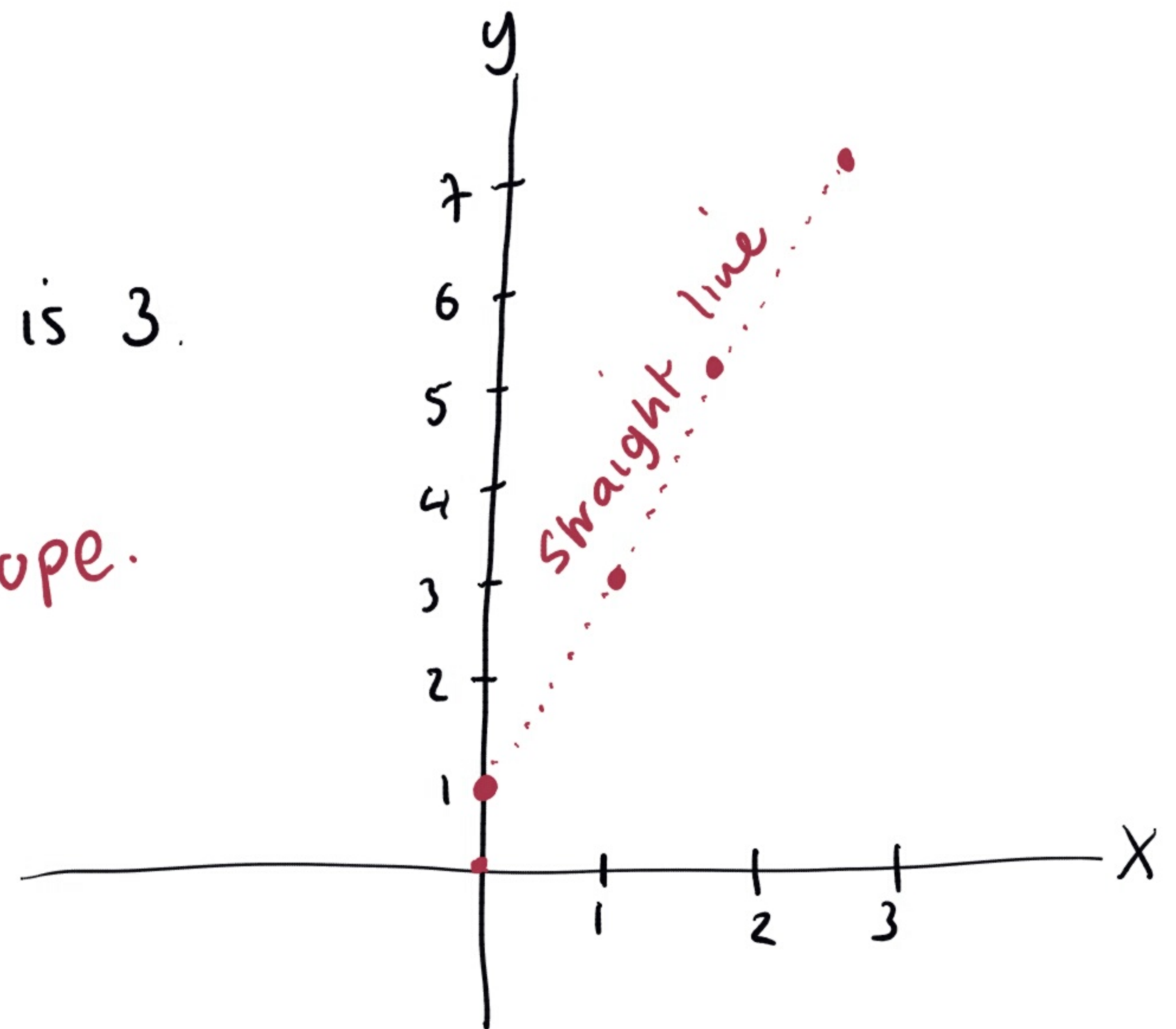
$$f(2) = 5$$

$$f(3) = 2(3) + 1$$

$$6 + 1$$

$$= 7$$

$$f(3) = 7$$

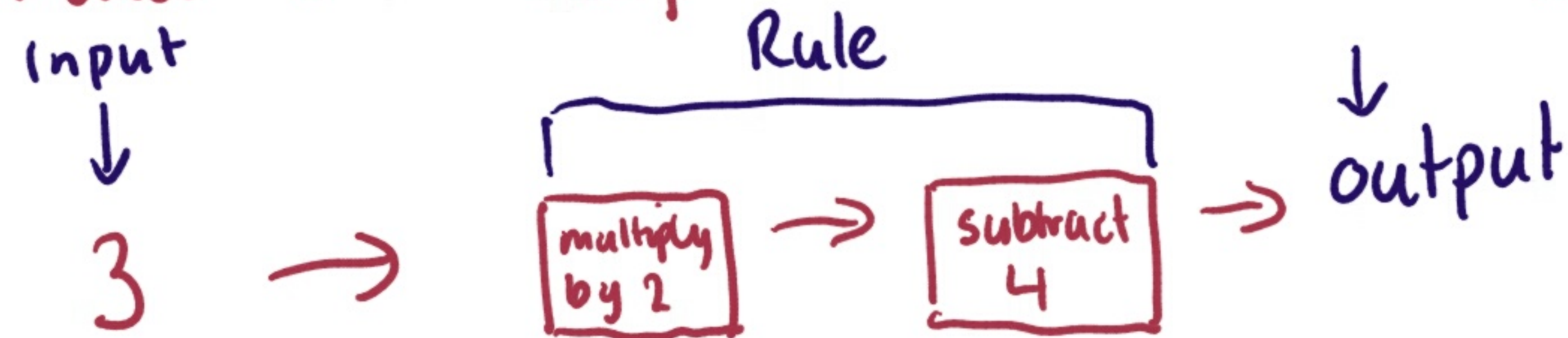


$$f(0) = 2(0) + 1$$
$$f(0) = 1$$

C/W  
Pg 455  
Q1, (11)  
Q2 +3

Input values x values Domain	Outputs values y values Range.	(x, y) point couple.
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Eg Find the output of the following



$$f(x) \rightarrow 2x - 4 =$$

$$f(3) \rightarrow 2(3) - 4 = 2$$

$$f(7) \rightarrow 2(7) - 4 = 10$$

$$f(8) \rightarrow 2(8) - 4 = 12$$

$$f(x) = y$$
$$f(3) = 2$$

$$f(7) = 10$$

$$f(8) = 12$$

Point, couple  
(3, 2)

(7, 10)

(8, 12)

Q<sub>1</sub>(ii)

$$f(x) = 2(x + 4)$$

$$f(2) = 2((2) + 4) \quad f(2) = 12 \quad (2, 12)$$

$$2(6)$$

$$= 12$$

$$f(1) = 2((1) + 4) \quad f(1) = 10 \quad (1, 10)$$

$$2(5)$$

$$= 10$$

$$f(0) = 2((0) + 4) \quad f(0) = 8 \quad (0, 8)$$

$$2(4)$$

$$= 8$$

$$Q_2) \quad x \quad by \quad 2$$

$$f(x) = 2x + 4$$

$$ii) \quad \frac{f(x)}{y} = 8x - 7$$

$$iii) \quad f(x) = \frac{x}{4} - 3$$

$$iv) \quad f(x) = 4(x + 3)$$

$$Q_3) \quad f(x) = 3x + 2$$

$$f(x) = 5x - 2$$

$$f(x) = \frac{x}{3} + 2$$

$$f(x) = 7(x + 2)$$

Pg 456 Q4

Input $x$	Rule $x^2 + 4$	Output $y$	Couple $(x, y)$
-2	$(-2)^2 + 4$	8	$(-2, 8)$
-1	$(-1)^2 + 4$	5	$(-1, 5)$
0	$(0)^2 + 4$	4	$(0, 4)$
1	$(1)^2 + 4$	5	$(1, 5)$
2	$(2)^2 + 4$	8	$(2, 8)$

Calculator

$x$	$f(x)$
-2	8
-1	5
0	4
1	5
2	8

4  
ii

Input $x$	Rule $3 - 2x$	Output $y$	Couple $(x, y)$
-3	$3 - 2(-3)$	9	$(-3, 9)$
-2	$3 - 2(-2)$	7	$(-2, 7)$
-1	$3 - 2(-1)$	5	$(-1, 5)$
0	$3 - 2(0)$	3	$(0, 3)$
1	$3 - 2(1)$	1	$(1, 1)$
2	$3 - 2(2)$	-1	$(2, -1)$

input $x$	$y$ $f(x)$
-3	9
-2	7

Q5)

$$\begin{array}{l} 3 \rightarrow \boxed{2(x)} \rightarrow \boxed{+ \square} \rightarrow 5 \leftarrow 2 \\ 2 \rightarrow \boxed{2(x)} \rightarrow \boxed{- \square} \rightarrow 3 \leftarrow 2 \\ 1 \rightarrow \boxed{2(x)} \rightarrow \boxed{- \square} \rightarrow 1 \leftarrow 2 \end{array}$$

$$\begin{array}{l} 2(3) \pm \square = 5 \quad 2(2) \pm \square = 3 \\ 6 - 1 = 5 \quad 4 - 1 = 3 \end{array}$$

Rule  
 $f(x) = 2x - 1$

Q5 ii)

$$\begin{array}{l} 0 \rightarrow \boxed{x3} \rightarrow \square \rightarrow 1 \\ 1 \rightarrow \boxed{x3} \rightarrow \square \rightarrow 4 \\ 5 \rightarrow \boxed{x3} \rightarrow \square \rightarrow 6 \end{array}$$

Rule  
 $f(x) = 3x + 1$

$$\begin{array}{l} 3(0) \pm \square = 1 \\ + 1 = +1 \end{array}$$

$$\begin{array}{l} 3(1) \pm \square = 4 \\ 3 + 1 = 4 \end{array}$$



Q6) Input  $x \rightarrow$   $[2x] =$   $[+1] \rightarrow$  Output =  $y$ .

$x \rightarrow$   $[2x] =$   $[+1] \rightarrow$  11

$x \rightarrow$   $[2x] =$   $[+1] \rightarrow$  17

$x \rightarrow$   $[2x] =$   $[+1] \rightarrow$  41

Make an equation.

$2x + 1 = 11$  solve for  $x$        $2x + 1 = 17$

$-1 \mid 2x = 10 \mid -1$        $-1 \mid 2x = 16 \mid -1$

$\div 2 \mid x = 5 \mid \div 2$        $\div 2 \mid x = 8 \mid \div 2$

6ii)  $x \rightarrow$   $[3x] \rightarrow$   $[-1] \rightarrow$  11

$x \rightarrow$   $[3x] \rightarrow$   $[-1] \rightarrow$  23

$x \rightarrow$   $[3x] \rightarrow$   $[-1] \rightarrow$  29

$3x - 1 = 11$

$+1 \mid 3x = 12 \mid +1$

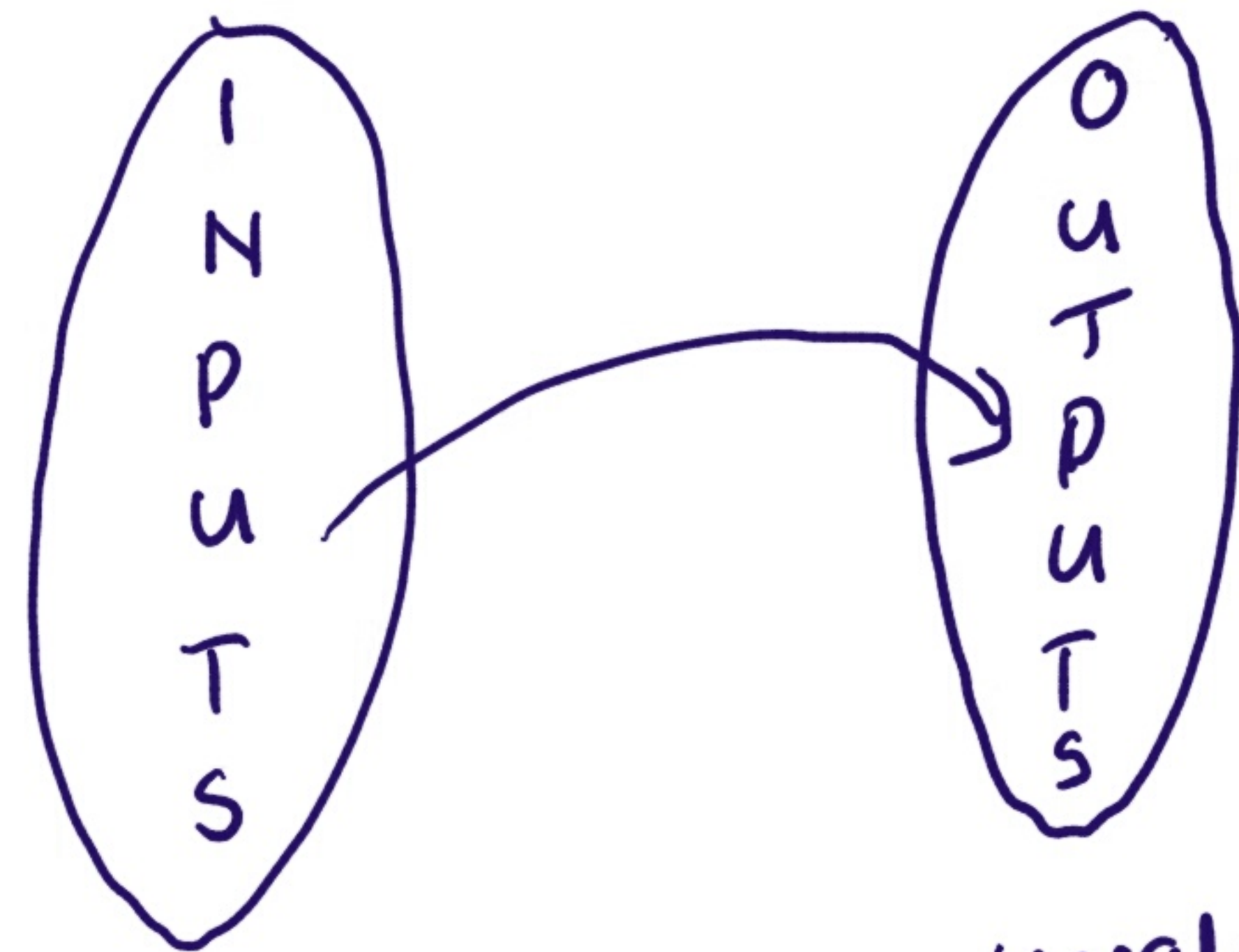
$x = 4$

$3x - 1 = 23$

$+1 \mid 3x = 24 \mid +1$

$x = 8$

# Mapping Diagram



x values

y values

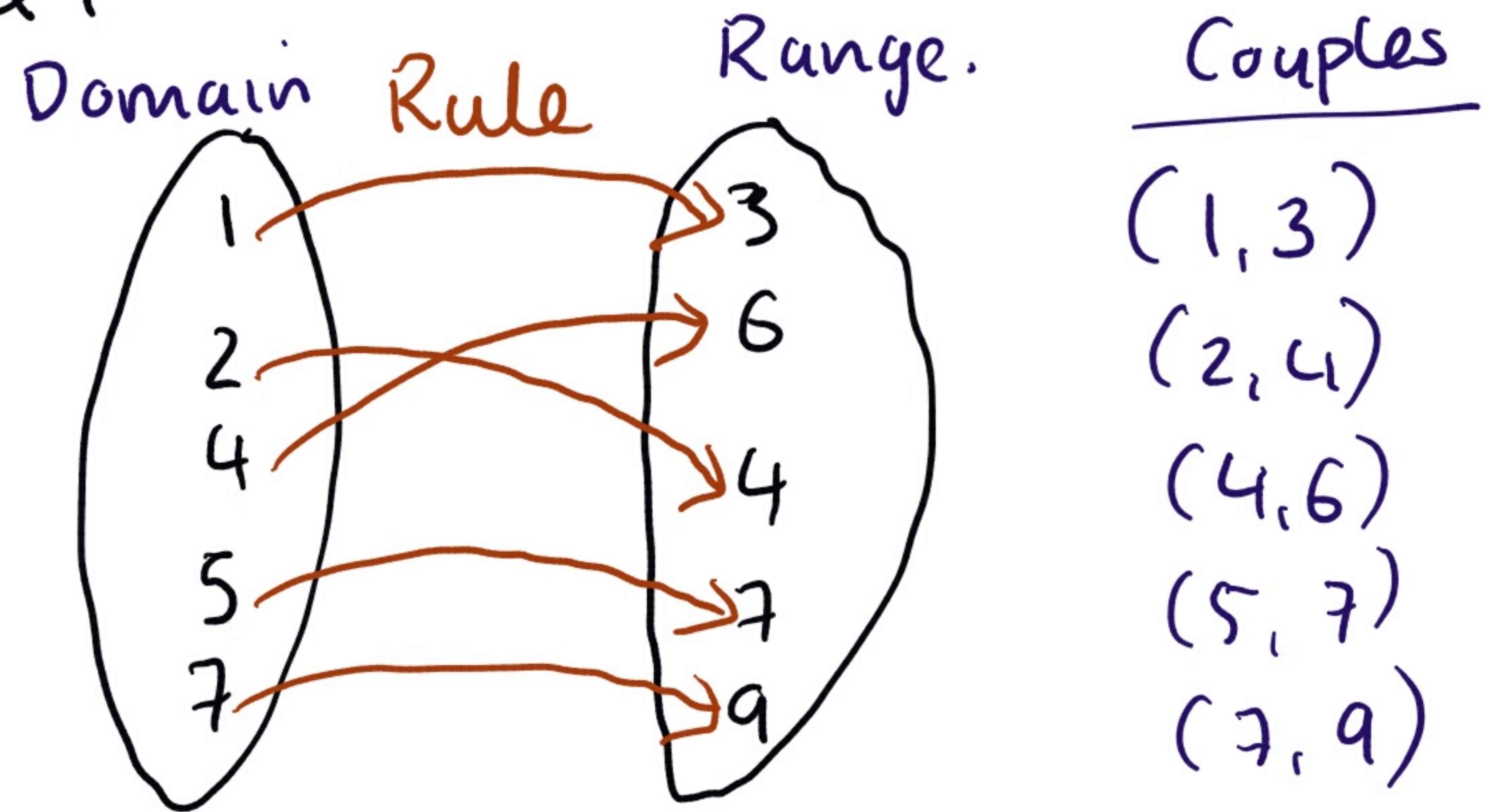
Domain

Range

Couple (Domain, Range)  
 (x, y)  
 (input, output)

In a function, one input can only have one output.

Q7



Rule

$$f(x) = x + 2$$

$$f(1) = (1) + 2$$

$$f_1 = 3$$

$$f(2) = (2) + 2$$

$$f_2 = 4 \checkmark$$

(13)

$$2x + 4 = 8$$
$$-4 \mid 2x = 4 \mid -4$$
$$x = 2$$

$$2x + 4 = -8$$
$$-4 \mid 2x = -12 \mid -4$$
$$x = -6$$

Hlw pg 458 Q 14 + 15,

Q14

$$f(x) \rightarrow 6x - 2$$

$$\begin{array}{l} \begin{matrix} x & y \\ (2, & a) \end{matrix} \Rightarrow 6(2) - 2 \\ 12 - 2 = 10 \\ (2, 10) \end{array}$$

$$\begin{array}{l} \begin{matrix} x & y \\ (-4, & b) \end{matrix} \Rightarrow 6(-4) - 2 \\ -24 - 2 = -26 \\ (-4, -26) \end{array}$$

$$\begin{matrix} x & y \\ (c, & 16) \end{matrix}$$

$$16 = 6x - 2$$

$$\begin{array}{l} +2 \quad | \quad 18 = 6x \quad | +2 \\ \div 6 \quad | \quad 3 = x \quad | \div 6 \end{array}$$

$$(3, 16)$$

$$\begin{matrix} x & y \\ (d, & -14) \end{matrix}$$

$$-14 = 6x - 2$$

$$\begin{array}{l} +2 \quad | \quad -12 = 6x \quad | +2 \\ \div 6 \quad | \quad -2 = x \quad | \div 6 \end{array}$$

$$(-2, -14)$$

$$1) y = 2x - 4$$

$$\begin{aligned} & 2(3) - 4 \\ & 6 - 4 \\ & = 2 \end{aligned}$$

$$2) y = 3x + 2$$

$$\begin{aligned} & 3(1) + 2 \\ & 3 + 2 = 5 \end{aligned}$$

$$3) y = 3x + 1$$

$$\begin{aligned} & 3(1) + 1 \\ & 3 + 1 = 4 \end{aligned}$$

# Operations with functions

## Method

① If  $f(x)$  is written with a value in the bracket i.e.  $f(1)$  replace the  $x$  value in the function with the value in the bracket

Eg 1) If  $f(x) = 2x - 3$ , find  $f(2)$

$$f(2) = 2(2) - 3$$
$$4 - 3$$

$$f(2) = 1$$

As a couple

$$f(2) = 1$$

$$\begin{matrix} x & y \\ (2, 1) \end{matrix}$$

find  $f(1) = 2(1) - 3$   
 $= -1$

$f(0) = 2(0) - 3$   
 $= -3$

$f(-3) = 2(-3) - 3$   
 $= -6 - 3$   
 $= -9$

## Method 2

When  $f(x) = \text{value}$ , you will have to put the function equal to the given value and solve for  $x$ .

Eg 1)  $f(x) = \underline{3x-2}$   
Find  $f(x) = 4$  given value.

$$\begin{array}{l|l} +2 \\ \hline 3x - 2 = 4 \\ +2 \\ \hline 3x = 6 \\ \div 3 \\ \hline x = 2 \end{array}$$

$$f(2) = 4$$

$(\overset{x}{2}, \overset{y}{4})$

Find  $f(x) = 7$

$$\begin{array}{l|l} +2 \\ \hline 3x - 2 = 7 \\ +2 \\ \hline 3x = 9 \\ \div 3 \\ \hline x = 3 \end{array}$$

If  $f(x) = 3x - 2$  and

$$g(x) = \underline{2 - 4x}$$

find  $g(x) = -10$

$$+2 - 4x = -10$$

$$\begin{array}{l|l} -2 \\ \hline -4x = -12 \\ \div -4 \\ \hline x = 3 \end{array}$$

$$g(3) = -10$$

Q1)  $g(x) = f(4)$

$$2 - 4x = 3(4) - 2$$

$$2 - 4x = 10$$

$$\begin{array}{l|l} -2 \\ \hline -4x = 8 \\ \div -4 \\ \hline x = -2 \end{array}$$

Pg 460 Q2+3



If  $f(x) = 2x^2 - 1$  and  $g(x) = x + 2$  are two functions

quadratic U shape      linear



find

①  $f(x) = \underline{3}$

$$2x^2 - 1 = 3$$

Solve for  $x$

$$\begin{array}{l} +1 \\ \hline \div 2 \\ \hline \sqrt{\quad} \end{array} \quad \begin{array}{l} 2x^2 = 4 \\ \div 2 \\ \hline \sqrt{\quad} \end{array}$$

$$x^2 = 2$$

$$x = \sqrt{2}$$

$$[-2]^2 = 4$$

②  $g(x) = f(\underline{3})$

$$x + 2 = 2(3)^2 - 1$$

$$x + 2 = 17$$

$$\begin{array}{l} -2 \\ \hline x = 15 \end{array} \quad \begin{array}{l} -2 \\ \hline \end{array}$$

③  $f(x) = g(x)$

$$2x^2 - 1 = x + 2$$

$$\begin{array}{l} -x \\ \hline 2x^2 - x - 1 = 2 \\ -2 \\ \hline 2x^2 - x - 3 = 0 \end{array}$$

mult  $6x^2$        $6N=6$   
 $2 \times 3$

$$2x^2 + 2x - 3x - 3$$

$$2x(x+1) - 3(x+1) = 0$$

$$(2x-3)(x+1) = 0$$

$$\begin{array}{l} 2x = 3 \\ x = 3/2 \end{array}$$

$$x = -1$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$2x - 3 = 0 \quad x + 1 = 0$$

$$2x = 3 \quad x = -1$$

$$x = 3/2$$

When you have to put a function into another function

Eg 1)  $f(x) = 2x - 3$  and  $g(x) = x + 5$  are two functions

① find  $f(2)$

$$2x - 3$$

$$2(2) - 3$$

$$4 - 3$$

$$f(2) = 1$$

②  $g(-2)$

$$x + 5$$

$$(-2) + 5$$

$$= 3$$

$$g(-2) = 3$$

③  $f(g(2))$

put the  $g$  function into the  $x$  part of the  $f$  function

$$f(x) = 2x - 3$$

$$g(x) = x + 5$$

$$2(x + 5) - 3$$

$$2x + 10 - 3$$

$$2x + 7$$

$$2(2) + 7$$

$$4 + 7$$

$$= 11$$

$$fg(2) = 11$$

④  $gf(-2)$

$$f(x) = 2x - 3$$

$$gf(-2)$$

put the f function  
into the x part  
of the g function

$$g(x) = x + 5$$

$$\begin{array}{c} f(x) \\ (2x - 3) + 5 \end{array}$$

$$2x - 3 + 5$$

$$2x + 2$$

$$\begin{aligned} gf(-2) &= 2(-2) + 2 \\ &= -4 + 2 \\ &= -2 \end{aligned}$$

HW pg 461 Q14