



T&T2h 19.4
Finding...



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Functions

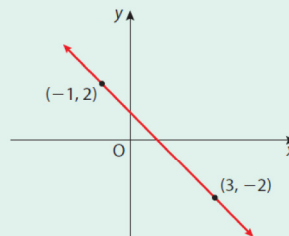
chapter
19

391

Section 19.4 Finding coefficients of functions

Example 1

The given diagram shows part of the graph of the function $y = ax + b$.
Find the values of a and b .



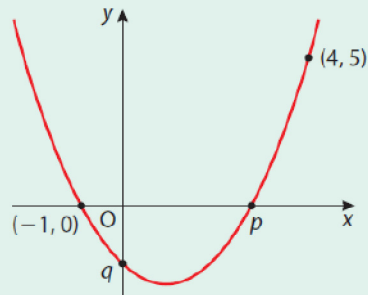
Example 2

The graph of the quadratic function

$$f(x) = x^2 + bx + c$$
 is shown.

Find the values of b and c .

Hence write down the coordinates of p and q .



Exercise 19.4

$$f(x) = y$$

1. $f(x) = 3x + k$ is a function.

If $f(4) = 10$, find the value of k .

$$\begin{array}{l} x = 4 \\ y = 10 \end{array} \left. \vphantom{\begin{array}{l} x = 4 \\ y = 10 \end{array}} \right\} \text{sub in to given function}$$

$$f(x) \\ 10 = 3(4) + k$$

$$10 = 12 + k$$

$$\begin{array}{l} -12 \\ \hline -2 = k \end{array} \quad \begin{array}{l} -12 \\ \hline \end{array}$$

$$\begin{array}{l} \text{verify} \\ \hline 3(4) - 2 = 10 \\ 12 - 2 = 10 \\ 10 = 10 \end{array}$$

2. If $(1, 5)$ is a couple of the function $f(x) = kx + 4$, find the value of k .

3. $f(x) = ax - 6$ is a function.
If $f(2) = -2$, find the value of a .

4. $f: x \rightarrow x^2 - 2x + k$ is a function.

If $(1, 2)$ is a couple of the function, find the value of k .

5. $(-3, 2)$ is a point on the line $y = ax + 11$. Find the value of a .

6. $f(x) = kx^2 + 3$ is a function.

If $(-1, -1)$ is a couple of this function, find the value of k .

7. The graph of the linear function

$$f(x) = ax + b$$

is shown. Find the values of a and b .

① $(0, 4)$ $x=0$
 $y=4$

$$4 = a(0) + b$$

$$4 = b$$

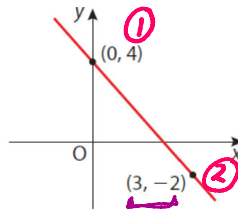
② $(3, -2)$ $x=3$ $b=4$
 $y=-2$

$$-2 = a(3) + 4$$

$$-2 = 3a + 4$$

$$\begin{array}{r|l} -4 & -6 = 3a \\ \div 3 & -2 = a \end{array}$$

$f(x)$



Note: Point (x, y)
sub the value for x into the function (x part)
Put the function equal to the y value

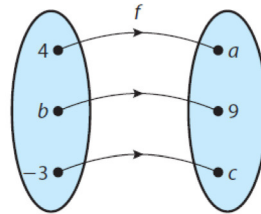
$$f(x) = ax + b$$

$$a = -2 \quad b = 4$$

$$f(x) = -2x + 4$$

8. A function f is defined as $f: x \rightarrow 2x - 1$.

If the mapping diagram on the right represents f , find the values of a , b and c .



9. $g: x \rightarrow ax^2 + bx + 1$ defines a function. \cup
 $a > 0$ \cap
 $-a < 0$
If $g(1) = 0$ and $g(2) = 3$, write down two equations in a and b .
Solve these equations to find the values of a and b .

① $x=1, y=0$ ② $x=2, y=3$

$$0 = a(1)^2 + b(1) + 1$$

$$3 = a(2)^2 + b(2) + 1$$

$$0 = a + b + 1$$

$$3 = 4a + 2b + 1$$

$$a + b + 1 = 0$$

$$4a + 2b + 1 = 3$$

$$-1 \mid a + b = -1 \mid -1$$

$$-1 \mid 4a + 2b = 2 \mid -1$$

Simultaneous Equation

$$\begin{array}{r} a + b = -1 \quad (-4) \\ 4a + 2b = 2 \end{array} \Rightarrow \begin{array}{r} -4a - 4b = 4 \\ \underline{4a + 2b = 2} \\ -2b = 6 \end{array}$$

$$\therefore b = -3$$

$$a + (-3) = -1$$

$$a - 3 = -1 \quad +3$$

$$a = 2$$

10. A function is defined by $f: x \rightarrow ax^2 + bx + 1$.

If $f(1) = 0$ and $f(-1) = 0$, find the value of a and the value of b .

① $x=1, y=0$ ② $x=-1, y=0$

$$a(1)^2 + b(1) + 1 = 0$$

$$a(-1)^2 + b(-1) + 1 = 0$$

$$a + b + 1 = 0$$

$$a - b + 1 = 0$$

$$-1 \mid a + b = -1 \mid -1$$

$$-1 \mid a - b = -1 \mid -1$$

$$\begin{array}{r} a + b = -1 \\ \underline{a - b = -1} \\ 2a = -2 \end{array} \therefore a = -1$$

$$a = -1 \Rightarrow a + b = -1$$

$$(-1) + b = -1$$

$$-1 + b = -1 \quad +1$$

$$b = 0$$

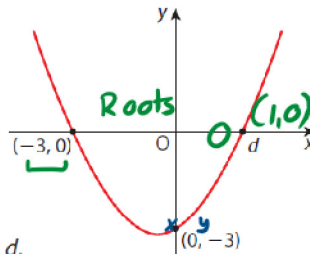
$a = -1$

11. $f: x \rightarrow x^2 + px + q$ defines a function.

Given that $f(3) = 4$ and $f(-1) = 4$, find the values of p and q .
Using these values for p and q , solve the equation $x^2 + px + q = 0$.

12. The function $f(x) = x^2 + bx + c$ is graphed on the right.

- (i) Use $f(0)$ to find the value of c .
- (ii) Use the graph to find another equation in b and c . Use this equation and the value for c found in (i) to find the value of b .
- (iii) Using these values for b and c , solve the equation $x^2 + bx + c = 0$ to find the coordinates of the point d .



i) $-3 = (0)^2 + b(0) + c$
 $-3 = c$

ii) $(-3, 0) \quad c = -3$
 $0 = (-3)^2 + b(-3) - 3$

$0 = 9 - 3b - 3$
 $0 = 6 - 3b$

$-6 \mid -6 = -3b \mid -6$
 $\div -3 \mid 2 = b \mid \div -3$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x + 3 = 0$	$x - 1 = 0$	$-1x$
$x = -3$	$x = 1$	$+3x$
$\underline{+3x}$		$+2x$

(x, y)

13. Functions f and g are defined as follows:

$f: x \rightarrow x^2 + 1$ and $g: x \rightarrow ax + b$, where a and b are constants.

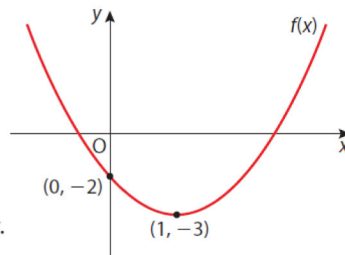
If $f(0) = g(0)$ and $g(2) = 15$, find the values of a and b .

14. The diagram shows part of the graph of the function

$$f: x \rightarrow x^2 + bx + c.$$

The named couples are elements of the function.

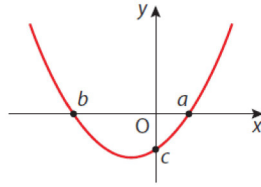
- (i) Find the values of b and c .
- (ii) If $(2, y)$ is a point on the graph, find the value of y .



15. The curve on the right is the graph of the function

$$y = x^2 + 2x - 3.$$

Find the coordinates of the points a , b and c .



16. $f(x) = 2x^2$ and $g(x) = 3x - 1$ are two functions.

Find (i) $f(3)$ (ii) $g(1)$ (iii) $g\left(\frac{1}{3}\right)$.

If $f(3) = kg(1)$, find k .

17. Given that $f(x) = 3^x$, find

- (i) $f(4)$ (ii) $f(-2)$ (iii) $f\left(\frac{1}{2}\right)$ (iv) the value of x for which $f(x) = \frac{\sqrt{3}}{3}$.

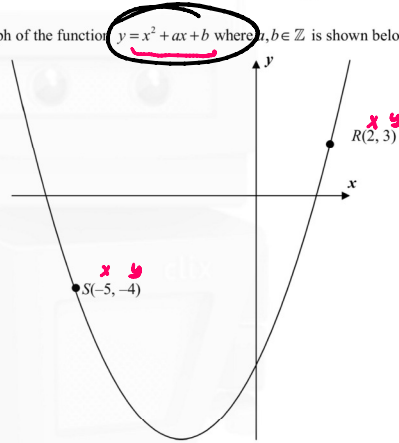
Answers

Exercise 19.4

1. -2 2. 1 3. 2
4. 3 5. 3 6. -4
7. $a = -2, b = 4$
8. $a = 7, b = 5, c = -7$
9. $a + b = -1, 2a + b = 1; a = 2, b = -3$
10. $a = -1, b = 0$
11. $p = -2, q = 1; x = 1$
12. (i) $c = -3$ (ii) $b = 2$ (iii) $(1, 0)$
13. $a = 7, b = 1$
14. (i) $b = -2, c = -2$ (ii) $y = -2$
15. $a(1, 0), b(-3, 0), c(0, -3)$
16. (i) 18 (ii) 2 (iii) $0; k = 9$
17. (i) 81 (ii) $\frac{1}{9}$ (iii) $\sqrt{3}$ (iv) $-\frac{1}{2}$

Question 11

(Suggested maximum time: 20 minutes)

Part of the graph of the function $y = x^2 + ax + b$ where $a, b \in \mathbb{Z}$ is shown below.The points $R(2, 3)$ and $S(-5, -4)$ are on the curve.(i) Use the given points to form two equations in a and b .

$$\begin{aligned} 2(4) + (-9) &= -1 \\ 8 - 9 &= -1 \\ -1 &= -1 \quad \checkmark \end{aligned}$$

$$\begin{aligned} R \quad x &= 2 \\ y &= 3 \\ 3 &= (2)^2 + a(2) + b \\ 3 &= 4 + 2a + b \\ -4 \quad | \quad -1 &= 2a + b \quad | \quad -4 \\ 2a + b &= -1 \end{aligned}$$

$$\begin{aligned} 2a + b &= -1 \quad (-1) \\ -5a + b &= -29 \end{aligned}$$

$$\begin{aligned} y &= x^2 + ax + b \\ S \quad x &= -5 \\ y &= -4 \\ -4 &= (-5)^2 + a(-5) + b \\ -4 &= 25 - 5a + b \\ -25 \quad | \quad -29 &= -5a + b \quad | \quad -25 \\ -5a + b &= -29 \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad -2a - b &= 1 \\ -5a + b &= -29 \\ \hline -7a &= -28 \\ \div 7 \quad a &= 4 \quad | \quad \div -7 \end{aligned}$$

$$\begin{aligned} 2(4) + b &= -1 \\ 8 + b &= -1 \\ -8 \quad | \quad -8 & \quad | \quad -8 \\ b &= -9 \end{aligned}$$