Finding a and d

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Method

1) Make TWO equations in terms of a and d.

2) Use simultaneous equations to find a value of a and d.

Eg 1 $T_4 = 11$ and $T_a = 21$ in an arithmetic sequence. Find the values of a and d.

$T_n = a + (n-1)d$

$T_4^n = 11$

$a + (4-1)d = 11$

$a + (3)d = 11$

(1) $a + 3d = 11$

$T_a^n = 21$

$a + (a-1)d = 21$

$a + (8)d = 21$

(2) $a + 8d = 21$

Simultaneous Equations Elimination method

(1) $a + 3d = 11$ (-1)

(2) $a + 8d = 21$ Multiply

$-a - 3d = -11$

$a + 8d = 21$

$5d = 10$

$\div 5 \mid d = 2 \mid \div 5$

$d = 2 \Rightarrow$ (1)

$a + 3d = 11$

$a + 3(2) = 11$

$a + 6 = 11$

$-6 \mid a = 5 \mid -6$

Q2 Pg 289

Q2 Pg 289

$T_4 = 14$ and $T_a = 34$ Find a and d

$T_n = a + (n-1)d$

$T_4^n = 14$

$a + (4-1)d = 14$

(1) $a + 3d = 14$

$T_a^n = 34$

$a + (a-1)d = 34$

(2) $a + 8d = 34$

Simultaneous Equations

(1) $a + 3d = 14$ $\times (-1)$

(2) $a + 8d = 34$

$\Rightarrow$

$-a - 3d = -14$

$a + 8d = 34$

$+5d = 20$

$\div 5 \mid d = 4 \mid \div 5$

$d = 4 \Rightarrow$ (1) $a + 3d = 14$

$a + 3(4) = 14$

$a + 12 = 14$

$T_n = a + (n-1)d$

$a = 2$

$d = 4$

$T_n = 2 + (n-1)4$

$2 + 4n - 4$

$T_n = 4n - 2$

$4+3(4)=14$

$a+12=14$

$-12 \mid a=2 \mid -12$

$T_n = 4n-2$

Homework Pg 289 Q3+4

T&T3 10.5

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chapter 10

Patterns and Sequences

Section 10.5 Finding the values of *a* and *d*

Example 1

T_4 of an arithmetic sequence is 11 and $T_9 = 21$.
Find the values of a and d and hence find T_{50}.

Example 2

If $x + 1$, $2x - 2$, and $2x + 1$ are three consecutive terms of an arithmetic sequence, find the value of x.
Hence write down T_n and T_{100} of the sequence.

Exercise 10.5

1. The first term of an arithmetic sequence is 5.
If the fifth term is 33, find d, the common difference.
Hence find T_n and T_{20}.

Exercise 10.5

2. In an arithmetic sequence, $T_4 = 14$ and $T_9 = 34$.
Find the values of a and d and hence write down the value of T_{13}.

Exercise 10.5

3. In an arithmetic sequence, $T_5 = 21$ and $T_{10} = 41$.
Find the values of a and d.
Hence find T_n and T_{60}.

$T_n = a + (n-1)d$

① $n=5 \Rightarrow a + (5-1)d = 21$
$a + 4d = 21$

② $n=10 \Rightarrow a + (10-1)d = 41$
$a + 9d = 41$

$a+4d=21$ multiply (-1) $\Rightarrow -a - 4d = -21$
$a + 9d = 41$
$5d = 20$
$\div 5 \mid d = 4 \mid \div 5$

$d=4 \rightarrow$ ① $a+4d=21$
$a+4(4)=21$
$a+16=21$
$-16 \mid a=5 \mid -16$
$a=5 \quad d=4$

$T_n = 5 + (n-1)4$ (a, d)
$5 + 4n - 4$
$T_n = 4n + 1$

$T_{60} = 4(60) + 1$
$240 + 1$
$T_{60} = 241$

Exercise 10.5

4. In an arithmetic sequence, the eighth term is -18 and the third term is 12.
Find the values of a and d.
Hence find T_{100}.

$T_n = a + (n-1)d$

$T_8 = -18$

(1) $a + (8-1)d = -18$

$a + 7d = -18$

$T_3 = 12$

(2) $a + (3-1)d = 12$

$a + 2d = 12$

$a + 2d = 12$ (-1) $\Rightarrow$ $-a - 2d = -12$

$a + 7d = -18$

$+5d = -30$

$\div 5 \mid d = -6 \mid \div 5$

$d = -6 \Rightarrow a + 2d = 12$

$a + 2(-6) = 12$

$a - 12 = 12$

$+12 \mid a = 24 \mid +12$

$T_n = 24 + (n-1)(-6)$

$24 - 6n + 6$

$T_n = -6n + 30$

$T_{100} = -6(100) + 30$

$-600 + 30$

$= -570$

Exercise 10.5

5. In an arithmetic sequence, $T_3 = 4$ and $T_{10} = -17$.
Find the values of a and d.
Write down T_n of the sequence and find the value of n for which $T_n = -47$.

$T_n = a + (n-1)d$

(1) $a + (3-1)d = 4$

$a + 2d = 4$

(2) $a + (10-1)d = -17$

$a + 9d = -17$

$a + 2d = 4$ (-1) $\Rightarrow$ $-a - 2d = -4$

$a + 9d = -17$

$+7d = -21$

$\div 7 \mid d = -3 \mid \div 7$

$d = -3 \Rightarrow a + 2d = 4$

$a + 2(-3) = 4$

$a - 6 = 4$

$+6 \mid a = 10 \mid +6$

$T_n = 10 + (n-1)(-3)$

$10 - 3n + 3$

$T_n = -3n + 13$

$-3n + 13 = -47$

$-13 \mid -3n = -60 \mid -13$

$\div -3$

$n = 20$

Exercise 10.5

6. In an arithmetic sequence, the first term is 3 and $T_6 = 2T_3$.

(i) Find the value of the common difference, d.

(ii) Find T_n, the nth term.

$T_n = a + (n-1)d$.

$T_1 = 3 \Rightarrow a + (1-1)d = 3$

$a + (0)d = 3$

$a = 3$

$T_6 = 2T_3$

$a + (6-1)d = 2(a + (3-1)d$

Exercise 10.5

7. In an arithmetic sequence, $T_1 + T_5 = 0$ and $T_{13} = 20$.

(i) Find the value of a and the value of d.

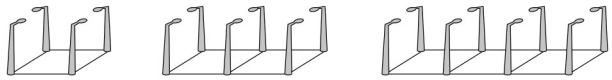
(ii) Show that the seventh term is twice the fifth term.

Exercise 10.5

8. In an arithmetic sequence, $T_4 = -9$ and $T_{15} = -31$.
Find the values of a and d.
Write down T_n of the sequence and hence find which term is equal to -81.

Exercise 10.5

9. Lamp-posts are put at the end of every 100 m stretch of a motorway, as shown,



(i) How many lamp-posts are needed for 500 m of motorway?
(ii) Write down, as a number sequence, the number of lamp-posts required for 100 m, 200 m, 300 m, 400 m, … .
(iii) Find an expression in n for the nth term of this sequence.
(iv) Use the expression found in (iii) to write down the number of lamp-posts needed for 8 km of motorway.
(v) The M51 is a motorway being built.
The contractor has ordered 2402 lamp-posts.
How long is this motorway?

Exercise 10.5

10. In an arithmetic sequence, $T_1 + T_3 = 12$ and $T_4 + T_6 = 24$.
Find the values of a and d.

Exercise 10.5

11. In an arithmetic sequence, the sixth term is 20 and the tenth term is four times the second term.
Find the values of a and d. Hence calculate T_{100}.

Exercise 10.5

12. If $x, 2x + 3$ and $4x + 5$ form three consecutive terms of an arithmetic sequence, find the value of x.

Exercise 10.5

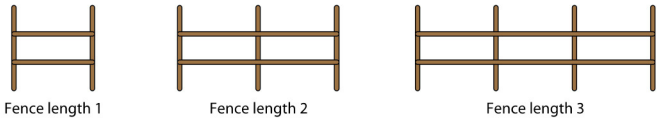
13. Find the value of x in each of the following arithmetic sequences:

(i) $x - 1, x + 1, 3x - 3$

(ii) $x + 4, 3 - x, x + 10$.

Exercise 10.5

14. Tommy builds fences in different lengths using pieces of wood.



Fence length 1 Fence length 2 Fence length 3

(i) Sketch fence length 5.
Tommy counted how many pieces he needed to make each fence length.
He then drew up the table below.

Fence length	1	2	3	4	5	6
Number of pieces	4	7	10			

(ii) Complete the table to show how many pieces of wood he would use for fence lengths 4, 5 and 6.

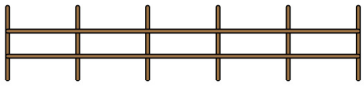
(iii) Write down, in terms of n, an expression for the number of pieces of wood needed for fence length n.

(iv) How many pieces of wood are needed for fence length 40?

(v) If 91 pieces of wood are needed, what is the number of the fence length?

Answers 10.5

1. $d = 7$; $T_n = 7n - 2$; $T_{20} = 138$
2. $a = 2$, $d = 4$; $T_{13} = 50$
3. $a = 5$, $d = 4$; $T_n = 4n + 1$; $T_{60} = 241$
4. $a = 24$, $d = -6$; $T_{100} = -570$
5. $a = 10$, $d = -3$; $T_n = -3n + 13$; $n = 20$
6. (i) $d = 3$ (ii) $T_n = 3n$
7. (i) $a = -4$, $d = 2$ (ii) Both = 8
8. $a = -3$, $d = -2$; $T_n = -2n - 1$; $T_{40} = 81$
9. (i) 12 (ii) 4, 6, 8, 10, …
(iii) $2n + 2$ (iv) 162 (v) 120 km
10. $a = 4$, $d = 2$
11. $a = 5$, $d = 3$; $T_{100} = 302$
12. $x = 1$
13. (i) $x = 3$ (ii) $x = -2$
14. (i)



(ii)

Fence length	1	2	3	4	5	6
No. of pieces	4	7	10	13	16	19

(iii) $3n + 1$ (iv) 121
(v) Fence length 30