



**PROJECT MATHS**

# Text & Tests

Leaving **3** Certificate

## Coordinate Geometry – The Line

chapter

3

*Key words*

Cartesian plane   origin   axis   quadrant   vertex   horizontal  
vertical   slope   parallel   perpendicular   positive   negative  
linear equation   area   translation   intersection   collinear

**Section 3.7 Parallel and perpendicular lines** \_\_\_\_\_

### Section 3.7 Parallel and perpendicular lines

If we are given the equation of a line  $\ell$ , such as  $2x + 3y - 4 = 0$ , we can find the slope of the line by expressing the equation in the form  $y = mx + c$ .

If we are also given a point  $(x_1, y_1)$ , we can then find the equation of a line through  $(x_1, y_1)$  and which is parallel to or perpendicular to  $\ell$ .

#### Example 1

Find the equation of the line through the point  $(-2, 3)$  which is perpendicular to the line  $2x - y + 5 = 0$ .

To find the slope of  $2x - y + 5 = 0$ , we express it in the form  $y = mx + c$ .

$$\begin{aligned} 2x - y + 5 &= 0 \\ \Rightarrow -y &= -2x - 5 \\ \Rightarrow y &= 2x + 5 \dots \quad \text{multiply each term by } -1 \\ \Rightarrow \text{the slope is } 2. \end{aligned}$$

The slope of the line perpendicular to this line is  $-\frac{1}{2}$ .

Equation of line through  $(-2, 3)$  with slope  $-\frac{1}{2}$  is:

$$\begin{aligned} y - y_1 &= m(x - x_1) & (x_1, y_1) &= (-2, 3) \\ y - 3 &= -\frac{1}{2}(x + 2) & m &= -\frac{1}{2} \\ y - 3 &= \frac{-x}{2} - 1 \\ \Rightarrow 2y - 6 &= -x - 2 \dots \quad \text{multiply each term by } 2 \\ \Rightarrow x + 2y - 4 &= 0 \text{ is the required equation.} \end{aligned}$$

Exercise 3.7

1. Find the slope of the line  $2x + y - 4 = 0$ .  
 Now find the equation of the line through the point  $(2, 4)$  and which is parallel to the line  $2x + y - 4 = 0$ .

Slope

$$2x + y - 4 = 0$$

$$\begin{array}{l|l} -2x & y - 4 = -2x \\ +4 & \end{array} \begin{array}{l} -2x \\ +4 \end{array}$$

$$y = -2x + 4$$

$$y = mx + c$$

$$m = -2 \text{ slope}$$

$y = mx + c$

Equation of a line

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - 2)$$

$$y - 4 = -2x + 4$$

$$y = mx + c$$

$$\star y = -2x + 8$$

OR

$$\star 2x + y - 8 = 0$$

Answer.

Exercise 3.7

2. Find the equation of the line through the point  $(1, -6)$  and which is parallel to the line  $3x - y + 4 = 0$ .

$y = mx + c$

$$\begin{array}{l|l} +y & 3x + 4 = y \\ & \end{array} \begin{array}{l} +y \\ \end{array}$$

$$m = 3$$

Slope

$x_1 \ y_1$

$$y - y_1 = m(x - x_1)$$

$$y + 6 = 3(x - 1)$$

$$y + 6 = 3x - 3$$

$$y = mx + c$$

$$\star y = 3x - 9$$

$$\star 3x - y - 9 = 0.$$

Exercise 3.7

3. Find the slope of the line  $2x - 3y + 1 = 0$ .

What is the slope of any line perpendicular to  $2x - 3y + 1 = 0$ ?

Now find the equation of the line through the point  $(4, -1)$  and which is perpendicular to the line  $2x - 3y + 1 = 0$ .

$$2x - 3y - 1 = 0$$

$$y = mx + c$$

$$\begin{array}{r|l} +3y & 2x - 1 = 3y \\ \hline \div 3 & \frac{2x - 1}{3} = y \end{array}$$

HIW pg 65  
Q4+5

$$\frac{2}{3} \perp \frac{-3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-3}{2}(x - 4)$$

$$2y + 2 = -3x + 12$$

$$y = mx + c$$

$$2y = -3x + 10$$

$$y = \frac{-3x + 10}{2}$$

$$\text{or } 3x + 2y - 10 = 0$$

65

Exercise 3.7

4. Find the equation of the line through  $(-2, 1)$  and which is perpendicular to the line  $3x + 2y - 4 = 0$ .

Slope  $y = mx + c$

$$3x + 2y - 4 = 0$$

$$\begin{array}{r|l} -3x & 2y - 4 = -3x \\ \hline +4 & 2y = -3x + 4 \\ \div 2 & y = \frac{-3x + 4}{2} \end{array}$$

$$\text{Slope} = \frac{-3}{2} \perp \frac{2}{3}$$

Equation of line  
 $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{2}{3}(x + 2)$$

$$3y - 3 = 2x + 4$$

$$y = mx + c$$

$$3y = 2x + 7$$

$$y = \frac{2x + 7}{3}$$

$$2x - 3y + 7 = 0$$

65

Exercise 3.7

5. Find the equation of the line through  $(-4, 0)$  and which is parallel to the line  $y = 3x - 5$ .

Slope  $y = mx + c$   
 $y = 3x - 5$   
 $m = 3$

$y - y_1 = m(x - x_1)$   
 $y - 0 = 3(x + 4)$   
 $y = 3x + 12$   
 $3x - y + 12 = 0$

65

Exercise 3.7

6. A line passes through the origin and is perpendicular to the line whose equation is  $3x - y - 2 = 0$ . Find the equation of the line.

$3x - 2 = y$   
 Slope = 3  $\perp$   $m$   
 $\frac{3}{1} \perp -\frac{1}{3}$

$y - y_1 = m(x - x_1)$   
 $y - 0 = -\frac{1}{3}(x - 0)$   
 $3y = -1x$   
 $y = -\frac{1}{3}$  OR  $x + 3y = 0$

65

Exercise 3.7

7. The point A has coordinates  $(1, 7)$  and the point B has coordinates  $(3, 1)$ .  
 The midpoint of [AB] is P.  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$   
 Find the coordinates of P.  
 Now find the equation of the line which passes through P and which is perpendicular to the line  $x + 5y - 7 = 0$ .

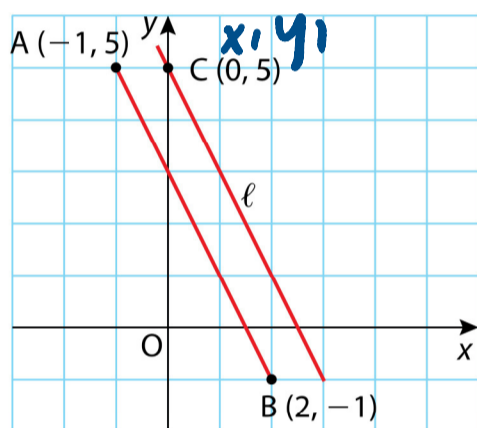
midpoint  $\left(\frac{1+3}{2}, \frac{7+1}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right) = P(2, 4)$

Slope  $x + 5y - 7 = 0$   
 $y = mx + c$   
 $5y - 7 = -x$   
 $5y = -x + 7$   
 $y = \frac{-1x + 7}{5}$   
 slope  $-1/5 \perp 5^m$

$y - y_1 = m(x - x_1)$   
 $y - 4 = 5(x - 2)$   
 $y - 4 = 5x - 10$   
 $y = 5x - 6$   
 $5x - y - 6 = 0$

Exercise 3.7

8. The given diagram shows the points A(-1, 5), B(2, -1) and C(0, 5).  
 The line  $\ell$  is parallel to AB and contains the point C.  
 Find the equation of  $\ell$ .



Slope of AB  
 $\frac{y_2 - y_1}{x_2 - x_1}$  OR  $\frac{\text{Rise}}{\text{Run}}$

$y - y_1 = m(x - x_1)$



Exercise 3.7

9. A line is perpendicular to the line whose equation is  $y = 4x - 3$ .  
Find the equation of the line if it crosses the  $y$ -axis at  $(0, 7)$ .

66

Exercise 3.7

10. Which one of the following lines is parallel to  $3x + y - 4 = 0$ ?

A:  $y = 3x - 2$

B:  $y = \frac{1}{3}x + 4$

C:  $6x + 2y + 7 = 0$

D:  $x + 3y + 2 = 0$

66

Exercise 3.7

11. The line  $y = 2x + 5$  intersects the  $y$ -axis at the point P.  
Find the equation of the line through P and which is perpendicular to  $y = 2x + 5$ .

Exercise 3.7

12. The equation of the line AB is  $5x - 3y = 26$ .
- (i) Find the slope of AB.
  - (ii) The point A has coordinates  $(4, -2)$  and a point C has coordinates  $(-6, 4)$ .
    - (a) Prove that AC is perpendicular to AB.
    - (b) Find the equation of the line AC, expressing your answer in the form  $ax + by = c$ .



### Exercise 3.7 Answers

1.  $-2; 2x + y - 8 = 0$
2.  $3x - y - 9 = 0$
3.  $\frac{2}{3}; -\frac{3}{2}; 3x + 2y - 10 = 0$
4.  $2x - 3y + 7 = 0$
5.  $3x - y + 12 = 0$
6.  $x + 3y = 0$
7.  $P(2, 4); 5x - y - 6 = 0$
8.  $2x + y - 5 = 0$
9.  $x + 4y - 28 = 0$
10. C
11.  $x + 2y - 10 = 0$
12. (i)  $\frac{5}{3}$       (ii) (b)  $3x + 5y = 2$

**Answers**