



PROJECT MATHS

Text & Tests

Leaving **3** Certificate

Coordinate Geometry – The Line

chapter

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Key words

Cartesian plane origin axis quadrant vertex horizontal
vertical slope parallel perpendicular positive negative
linear equation area translation intersection collinear

Section 3.7 Parallel and perpendicular lines _____

Section 3.7 Parallel and perpendicular lines

If we are given the equation of a line ℓ , such as $2x + 3y - 4 = 0$, we can find the slope of the line by expressing the equation in the form $y = mx + c$.

If we are also given a point (x_1, y_1) , we can then find the equation of a line through (x_1, y_1) and which is parallel to or perpendicular to ℓ .

Example 1

Find the equation of the line through the point $(-2, 3)$ which is perpendicular to the line $2x - y + 5 = 0$.

To find the slope of $2x - y + 5 = 0$, we express it in the form $y = mx + c$.

$$\begin{aligned} 2x - y + 5 &= 0 \\ \Rightarrow -y &= -2x - 5 \\ \Rightarrow y &= 2x + 5 \dots \quad \text{multiply each term by } -1 \\ \Rightarrow \text{the slope is } 2. \end{aligned}$$

The slope of the line perpendicular to this line is $-\frac{1}{2}$.

Equation of line through $(-2, 3)$ with slope $-\frac{1}{2}$ is:

$$\begin{aligned} y - y_1 &= m(x - x_1) & (x_1, y_1) &= (-2, 3) \\ y - 3 &= -\frac{1}{2}(x + 2) & m &= -\frac{1}{2} \\ y - 3 &= \frac{-x}{2} - 1 \\ \Rightarrow 2y - 6 &= -x - 2 \dots \quad \text{multiply each term by } 2 \\ \Rightarrow x + 2y - 4 &= 0 \text{ is the required equation.} \end{aligned}$$

Exercise 3.7

1. Find the slope of the line $2x + y - 4 = 0$.
 Now find the equation of the line through the point $(2, 4)$ and which is parallel to the line $2x + y - 4 = 0$.

Slope

$$2x + y - 4 = 0$$

$$\begin{array}{l|l} -2x & y - 4 = -2x \\ +4 & \end{array} \begin{array}{l} -2x \\ +4 \end{array}$$

$$y = -2x + 4$$

$$y = mx + c$$

$$m = -2 \text{ slope}$$

$y = mx + c$

Equation of a line

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - 2)$$

$$y - 4 = -2x + 4$$

$$y = mx + c$$

$$\star y = -2x + 8$$

OR

$$\star 2x + y - 8 = 0$$

Answer.

Exercise 3.7

2. Find the equation of the line through the point $(1, -6)$ and which is parallel to the line $3x - y + 4 = 0$.

$y = mx + c$

$$\begin{array}{l|l} +y & 3x + 4 = y \\ & \end{array} \begin{array}{l} +y \\ \end{array}$$

$$m = 3$$

Slope

$x_1 \ y_1$

$$y - y_1 = m(x - x_1)$$

$$y + 6 = 3(x - 1)$$

$$y + 6 = 3x - 3$$

$$y = mx + c$$

$$\star y = 3x - 9$$

$$\star 3x - y - 9 = 0.$$

Exercise 3.7

3. Find the slope of the line $2x - 3y + 1 = 0$.

What is the slope of any line perpendicular to $2x - 3y + 1 = 0$?

Now find the equation of the line through the point $(4, -1)$ and which is perpendicular to the line $2x - 3y + 1 = 0$.

$$2x - 3y - 1 = 0$$

$$y = mx + c$$

$$\begin{array}{l|l} +3y & 2x - 1 = 3y \\ \hline \div 3 & \frac{2x - 1}{3} = y \end{array}$$

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$$\frac{2}{3} \perp \frac{-3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-3}{2}(x - 4)$$

$$2y + 2 = -3x + 12$$

$$y = mx + c$$

$$2y = -3x + 10$$

$$\downarrow y = \frac{-3x + 10}{2}$$

$$\text{or } \downarrow 3x + 2y - 10 = 0.$$

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Exercise 3.7

4. Find the equation of the line through $(-2, 1)$ and which is perpendicular to the line $3x + 2y - 4 = 0$.

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Exercise 3.7

5. Find the equation of the line through $(-4, 0)$ and which is parallel to the line $y = 3x - 5$.

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Exercise 3.7

6. A line passes through the origin and is perpendicular to the line whose equation is $3x - y - 2 = 0$. Find the equation of the line.

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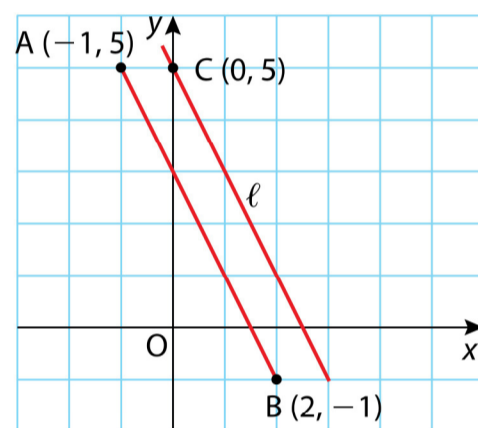
Exercise 3.7

7. The point A has coordinates (1, 7) and the point B has coordinates (3, 1).
The midpoint of [AB] is P.
Find the coordinates of P.
Now find the equation of the line which passes through P and which is perpendicular to the line $x + 5y - 7 = 0$.

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Exercise 3.7

8. The given diagram shows the points A(-1, 5), B(2, -1) and C(0, 5).
The line ℓ is parallel to AB and contains the point C.
Find the equation of ℓ .



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Exercise 3.7

9. A line is perpendicular to the line whose equation is $y = 4x - 3$.
Find the equation of the line if it crosses the y -axis at $(0, 7)$.

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Exercise 3.7

10. Which one of the following lines is parallel to $3x + y - 4 = 0$?

A: $y = 3x - 2$

B: $y = \frac{1}{3}x + 4$

C: $6x + 2y + 7 = 0$

D: $x + 3y + 2 = 0$

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Exercise 3.7

11. The line $y = 2x + 5$ intersects the y -axis at the point P.
Find the equation of the line through P and which is perpendicular to $y = 2x + 5$.

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Exercise 3.7

12. The equation of the line AB is $5x - 3y = 26$.
- Find the slope of AB.
 - The point A has coordinates $(4, -2)$ and a point C has coordinates $(-6, 4)$.
 - Prove that AC is perpendicular to AB.
 - Find the equation of the line AC, expressing your answer in the form $ax + by = c$.

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Exercise 3.7 Answers

1. $-2; 2x + y - 8 = 0$
2. $3x - y - 9 = 0$
3. $\frac{2}{3}; -\frac{3}{2}; 3x + 2y - 10 = 0$
4. $2x - 3y + 7 = 0$
5. $3x - y + 12 = 0$
6. $x + 3y = 0$
7. $P(2, 4); 5x - y - 6 = 0$
8. $2x + y - 5 = 0$
9. $x + 4y - 28 = 0$
10. C
11. $x + 2y - 10 = 0$
12. (i) $\frac{5}{3}$ (ii) (b) $3x + 5y = 2$

Answers