Arithmetic Series

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In a series all the terms are added

1, 3, 5, 7, 9..... sequence.

1+3+5+7+9 ... series.

Sum = added together

log tables pg22 $S_n = \frac{n}{2}[2a+(n-1)d]$

a = 1st term

d = common difference.

$a = 1$ $d = 2$

n = no. of terms

$S_5 = \frac{5}{2}[2(1)+(5-1)2]$

$S_5 = 25$

T&T3 10.6

T&T3 10.6.pptx

chapter 10 Patterns and Sequences

Section 10.6 Arithmetic series

Example 1

Find S_n and hence S_{20} of the series $5 + 8 + 11 + 14 + \ldots$.

Example 2

Given the arithmetic series $5 + 7 + 9 + \ldots$.
If $S_n = 192$, find the value of n.

Example 3

In an arithmetic series, $S_n = n^2 + 2n$.
Find S_1, S_2 and S_3 and hence write down T_1, T_2 and T_3.

Exercise 10.6

1. For the arithmetic series $2 + 5 + 8 + \ldots$,
   (i) find the value of a and d
   (ii) find the sum of the first 12 terms.

Exercise 10.6

2. Find the sum of the first 20 terms of the series
   $3 + 7 + 11 + 15 + \ldots$

Exercise 10.6

3. Find S_n and hence S_{16} of the arithmetic series

$1 + 4 + 7 + 10 + \ldots$

Exercise 10.6

4. The first four terms of a series are $7 + 10 + 13 + 16 + \ldots$

Find S_8, the sum of the first eight terms.

Exercise 10.6

5. Write down the value of a and the value of d for the series

$\overset{T_1}{16} + \overset{T_2}{12} + \overset{T_3}{8} + \overset{T_4}{4} + \ldots$

$-4 \quad -4$

Hence find S_{24} of the series.

$a = 16$

$d = -4$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{24} = \frac{24}{2}\left[2(16) + (24-1)-4\right]$

$S_{24} = -720$

Exercise 10.6

6. In an arithmetic series the nth term, $T_n = 5n - 2$.
Find the values of a and d and hence find S_{16} of the series.

$T_1 = 5(1) - 2$ $\quad T_1 = 3 \Rightarrow a$ first term

$5 - 2 = 3$

$T_2 = 5(2) - 2$ $\quad T_2 = 8$ $\quad 3, 8$

$10 - 2 = 8$ $\quad +5 = d$

$a = 3$

$d = 5$

$n = 16$

$S_{16} = \frac{16}{2}\underbrace{[2(3) + (16-1) \times 5]}_{\text{Calc}}$

$S_{16} = 648$

Exercise 10.6

7. Show that S_n of the series $1 + 2 + 3 + \ldots$ is $\frac{n}{2}(n + 1)$.
Hence find the sum of the series $1 + 2 + 3 + \ldots + 100$.

$S_n = \frac{n}{2}[2(1) + (n-1)1]$

$a = 1$

$d = 1$

$\frac{n}{2}[2 + n - 1]$

$\frac{n}{2}[n + 1]$

$n = 100$

$S_{100} = \frac{(100)}{2}[100 + 1]$

$S_{100} = 5050$

Exercise 10.6

8. S_n of the series $-4 - 2 + 0 + 2 + \ldots$ is 84.
(i) Write down the value of a and the value of d.
(ii) Find the value of n.

$a = -4$

$d = 2$

$S_n = \frac{n}{2}[2(-4) + (n-1)2]$

$\frac{n}{2}[-8 + 2n - 2]$

$S_n = \frac{n}{2}[2n - 10] = \frac{2n^2 - 10n}{2}$

$S_n = n^2 - 5n$

$n^2 - 5n = 84$

$-84 \mid n^2 - 5n - 84 = 0 \mid -84$

Factorize $n^2 - 5n - 84 = 0$

$(n + 7)(n - 12) = 0$

$-12n$

$+7n$

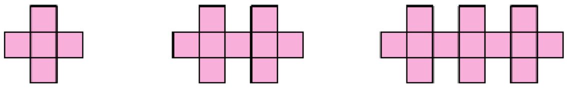
$-5n$

$n + 7 = 0 \quad \{ \quad n - 12 = 0$

$n = -7 \quad \quad n = +12$ ✓

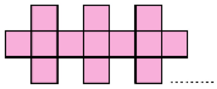
Exercise 10.6

9. Here are some patterns made of squares.



Pattern number 1 Pattern number 2 Pattern number 3

The diagram on the right shows part of Pattern number 4.



Pattern number 4

(i) Copy and complete Pattern number.

(ii) How many squares are there in Pattern 6? 25

(iii) Find an expression for the number of squares in Pattern n. $T_n = 4n - 1$

(iv) How many squares are there in total in the first 20 Patterns? Sum of series

$T_n = a + (n-1)d$ $a = 5$ $d = 4$

$5 + (n-1)4$

$5 + 4n - 4$

$T_n = 4n + 1$

$S_n = [2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(5) + (20-1)4]$

$S_{20} = 860$ squares.

Exercise 10.6

$T_n = a + (n-1)d.$

10. In an arithmetic series, $T_5 = 9$ and $T_8 = 27$.

(i) Find the values of a and d.

(ii) Find S_{10} of the series.

$T_5 = 9$

$a + (5-1)d = 9$

① $a + 4d = 9$

$T_8 = 27$

$a + (8-1)d = 27$

② $a + 7d = 27$

$a + 4d = 9$ (-1) $\Rightarrow$ $\cancel{-a} - 4d = -9$

$a + 7d = 27$ $\qquad \cancel{a} + 7d = 27$

$+3d = 18$

$\div 3 |$ $d = 6$ $| \div 3$

$d = 6 \Rightarrow a + 4d = 9$

$a + 4(6) = 9$

$a + 24 = 9$

$-24 |$ $a = -15$ $| -24$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$n = 10$

$a = -15$

$d = 6$

$S_{10} = \frac{10}{2}[\underbrace{2(-15) + (10-1)6}_{\text{calculator}}]$

$S_{10} = 120$

Exercise 10.6

11. In an arithmetic series, $T_3 = 0$ and $T_8 = 10$. $T_n = a + (n-1)d$

Find the values of a and d and hence find S_n of the series.

How many terms of the series must be added so that their sum is 36? S_n

$T_3 = 0$

$a + (3-1)d = 0$

① $a + 2d = 0$

$T_8 = 10$

$a + (8-1)d = 10$

② $a + 7d = 10$

$a + 2d = 0$ (-1) $\Rightarrow$ $\cancel{-a} - 2d = 0$

$a + 7d = 10$ $\qquad \cancel{a} + 7d = 10$

$+5d = 10$

$\div 5 |$ $d = 2$ $| \div 5$

$d = 2 \Rightarrow a + 2d = 0$

$a + 2(2) = 0$

$a + 4 = 0$

$a = -4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$a = -4$

$d = 2$

$S_n = \frac{n}{2}(2(-4) + (n-1)2]$

$= \frac{n}{2}(-8 + 2n - 2]$

$= \frac{n}{2}(2n - 10)$ $= \frac{2n^2 - 10n}{2}$

$= n^2 - 5n$

$n^2 - 5n = 36$

$-36 |$ $n^2 - 5n - 36 = 0$ $| -36$

Quadratic

Factorize

$(n - 9)(n + 4) = 0$

$+4n$

$-9n$

$-5n$

$n - 9 = 0$ $n = 9$ ✓

$n + 4 = 0$ $n = -4$

$(a)^2$ $5(a)$

Exercise 10.6

HW

12. Find S_n of the series $5 + 8 + 11 + 14 + \dots$ $\frac{n}{2}[\dots]$

$(9)^2 - 5(9)$

$81 - 45 = 36$

Exercise 10.6

H/W

12. Find S_n of the series $5 + 8 + 11 + 14 + \ldots$.
If $S_n = 98$, find the value of n.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$a = 5$
$d = 3$

$S_n = \frac{n}{2}[2(5)+(n-1)3]$

$\Rightarrow \frac{n}{2}[10+3n-3]$

$\Rightarrow \frac{n}{2}[3n+7] \Rightarrow \frac{3n^2+7n}{2}$

$S_n = \frac{3n^2+7n}{2} = 98$

$3n^2+7n = 196$

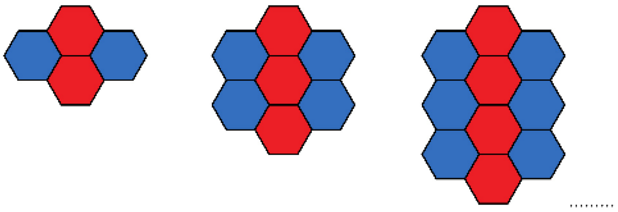
$3n^2+7n-196=0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Exercise 10.6

H/W

13. A student made tile designs using red and blue tiles, as shown.



(i) Find an expression in n for the total number of
 (a) red tiles used in the nth design
 (b) blue tiles used in the nth design.
(ii) Find, in terms of n, an expression for the total number of tiles used in the nth design.
(iii) How many tiles in total are needed to complete 10 designs using the same pattern?

Exercise 10.6

14. Which term of the series $3 + 8 + 13 \ldots$ is 98?
Now find the sum of these terms.

Exercise 10.6

15. S_n of an arithmetic series is given by $S_n = n^2 + 6n$.
Find S_1 and S_2 and hence write down the values of T_1 and T_2.

Exercise 10.6

16. (i) Write down the 10th term of the sequence which begins $3, 7, 11, 15, \ldots$
(ii) Write down an expression for the nth term of this sequence.
(iii) Show that 1997 cannot be a term in this sequence.
(iv) Calculate the number of terms in the sequence $3, 7, 11, 15, \ldots, 399$.
(v) Hence find the sum of the series $3 + 7 + 11 + 15 + \ldots + 399$.

Exercise 10.6

17. The nth term of an arithmetic series is $T_n = 52 - 4n$.
(i) Find the values of a and d.
(ii) Find which term is zero.
(iii) Find the sum of the terms which are positive.

Exercise 10.6

18. The sum of the first n terms of an arithmetic series is given by

$$S_n = 4n^2 - 8n.$$

(i) Use S_1 and S_2 to find the first term and the common difference.

(ii) How many terms of the series must be added to give a sum of 252?

Exercise 10.6

19. In an arithmetic series, $T_5 = 21$ and $T_{10} = 11$.

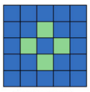
(i) Find the first term and the common difference.

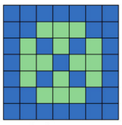
(ii) Find the sum of the first 20 terms.

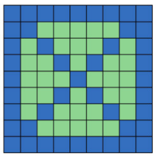
(iii) For what value of n is $S_n = 0$?

Exercise 10.6

20. The first three patterns of a tiling sequence are shown below.
The sequence continues in the same way.


1st pattern


2nd pattern


3rd pattern

In each pattern, the tiles form a square of blue and green tiles.

(i) In the table below, write down the number of blue tiles needed for each of the first five patterns.

Pattern	1	2	3	4	5
Number of blue tiles	21	33			

(ii) Find, in terms of n, an expression for the number of blue tiles needed for the nth pattern.

(iii) Use the formula for T_n, found in (ii) above, to find the number of blue tiles in the 10th pattern.

(iv) Find, in terms of n, a formula for the total number of blue tiles in the first n patterns.

(v) How many patterns can be made with 399 blue tiles?

Answers 10.6

1. (i) $a = 2, d = 3$ (ii) 222

2. 820

3. $S_n = \frac{n}{2}(3n - 1)$; 376

4. 140

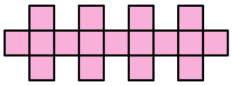
5. $a = 16, d = -4$; -720

6. $a = 3, d = 5$; $S_{16} = 648$

7. 5050

8. (i) $a = -4, d = 2$ (ii) $n = 12$

9. (i)



(ii) 25

(iii) $T_n = 4n + 1$ (iv) 860

10. (i) $a = -15, d = 6$ (ii) 120

11. (i) $a = -4, d = 2$ (ii) $S_n = n(n - 5)$; 9

12. $S_n = \frac{n}{2}(3n + 7)$; $n = 7$

13. (i) (a) $n + 1$ (b) $2n$

(ii) $T_n = 3n + 1$ (iii) 175

14. T_{20}; 1010

15. $S_1 = 7, S_2 = 16$; $T_1 = 7, T_2 = 9$

16. (i) 39 (ii) $T_n = 4n - 1$

(iv) 100 (v) 20 100

17. (i) $a = 48, d = -4$ (ii) T_{13}

(iii) 312

18. (i) $a = -4, d = 8$ (ii) 9

19. (i) $a = 29, d = -2$ (ii) 200

(iii) $n = 30$

20. (i)

Pattern	1	2	3	4	5
No. of blue tiles	21	33	45	57	69

(ii) $T_n = 12n + 9$ (iii) 129

(iv) $S_n = \frac{n}{2}(30 + 12n)$ (v) 7