Area of a triangle.

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T&T3 3.10

PROJECT MATHS

Text & Tests

Leaving 3 Certificate

chapter 3

Coordinate Geometry – The Line

Key words

Cartesian plane origin axis quadrant vertex horizontal
vertical slope parallel perpendicular positive negative
linear equation area translation intersection collinear

Section 3.10 Area of a triangle

1) One point has to be on the origin
2) Label the other two points (x_1, y_1) (x_2, y_2)
3) Sub the values into the formula.
Pg 16 Log Tables

Example 1

Find the area of the triangle with vertices (0, 0), (−2, 1) and (3, 4).

formula

$$\text{Area} = \tfrac{1}{2}|x_1 y_2 - x_2 y_1|$$

Answer can't be negative

(x_1, y_1) → $(-2, 1)$ $\quad$ (x_2, y_2) → $(3, 4)$

$$= \tfrac{1}{2}|(-2)(4) - (3)(1)|$$
$$= \tfrac{1}{2}|-8 - 3|$$
$$= \tfrac{1}{2}|-11|$$
$$= 5\tfrac{1}{2} \text{ square units}$$

Example 2

Find the area of the triangle with vertices (2, 4), (−3, 1) and (3, −5).

Let $(2, 4) \rightarrow (0, 0)$
$(-3, 1) \rightarrow (-5, -3)$
$(3, -5) \rightarrow (1, -9)$

Here we take 2 from each x-value and 4 from each y-value.

(x_1, y_1) → $(-5, -3)$ $\quad$ (x_2, y_2) → $(1, -9)$

$$\text{Area of triangle} = \tfrac{1}{2}|x_1 y_2 - x_2 y_1|$$
$$= \tfrac{1}{2}|(-5)(-9) - (1)(-3)|$$
$$= \tfrac{1}{2}|45 + 3|$$
$$= \tfrac{1}{2}|48|$$
$$= 24 \text{ square units}$$

Exercise 3.10 — Area of Triangle with (0,0) = ½|$x_1y_2 - x_2y_1$|

1. Find the area of the triangle whose vertices are

(i) (0, 0), (2, 1), (3, 4) — origin, $x_1\ y_1$, $x_2\ y_2$

(ii) (0, 0), (5, 1), (3, 6) — $x_1\ y_1$, $x_2\ y_2$

(iii) (0, 0), (−2, 3), (1, −4) — $x_1\ y_1$, $x_2\ y_2$

(iv) (0, 0), (3, 4), (−2, −6) — $x_1\ y_1$, $x_2\ y_2$

(v) (2, −1), (−2, 4), (0, 0)

(vi) (0, 0), (6, 0), (−2, 3).

iii $A = \frac{1}{2}|(-2)(-4) - (1)(3)|$

$\frac{1}{2}|8 - 3| = \frac{1}{2}|5|$

$5/2$ or $2.5u^2$

ii) $A = \frac{1}{2}|(5)(6) - (3)(1)|$ (x_1 y_2 x_2 y_1)

$\frac{1}{2}|30 - 3| = \frac{1}{2}|27|$

$= 13.5u^2$

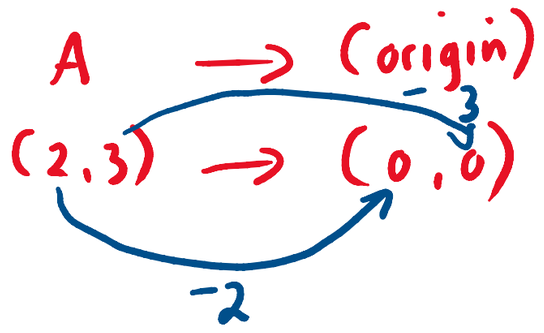
iv) $A = \frac{1}{2}|(3)(-6) - (-2)(4)|$

$\frac{1}{2}|-18 + 8|$

$\frac{1}{2}|-10|$ $\frac{1}{2}|10| = 5u^2$

Exercise 3.10 — Area of Triangle with (0,0) = ½|$x_1y_2 - x_2y_1$|

2. A(2, 3), B(−5, 1) and C(3, 1) are the vertices of a triangle.
By using the translation A(2, 3) → (0, 0), find the images of B and C under this translation. Hence find the area of the triangle ABC.



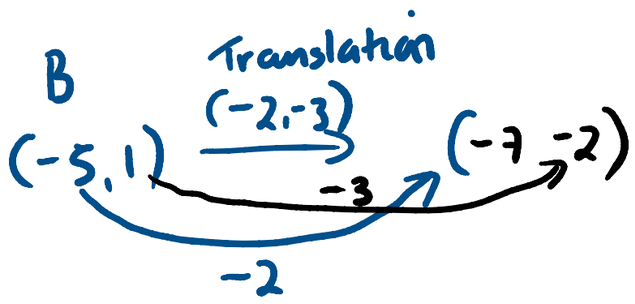
Translation $(-2, -3)$

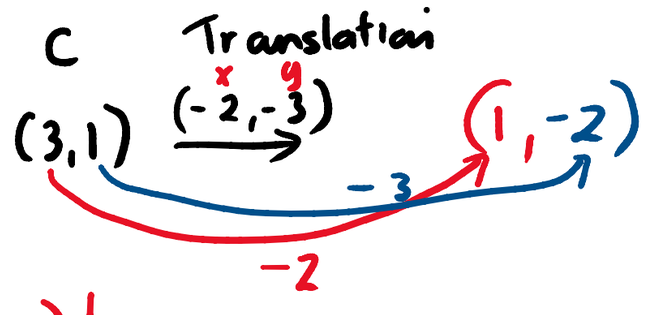
$A \to (0, 0)$

$B \to (-7, -2)$ (x_1 y_1)

$C \to (1, -2)$ (x_2 y_2)

Area of Δ ABC





Area = $\frac{1}{2}|(-7)(-2) - (1)(-2)|$

$\frac{1}{2}|14 + 2| \Rightarrow \frac{1}{2}|16| = 8$ units^2

Classwork
→ HW
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H/w.

3. By translating one of the vertices to (0, 0), find the area of each of the triangles whose vertices are

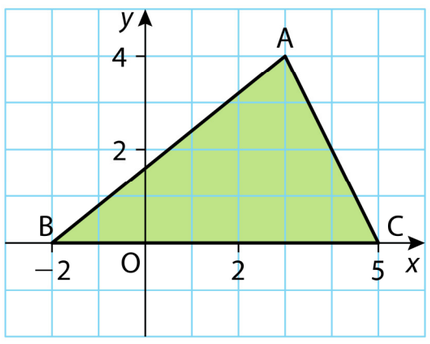
(i) (2, 3), (5, 1) and (2, 0)

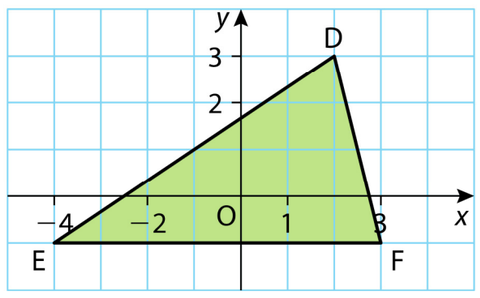
(ii) (−2, 3), (4, 0) and (1, −4)

(iii) (−2, 1), (3, 6) and (0, −3)

(iv) (5, 1), (2, −3) and (7, 1).

4. The area of a triangle is *half length of base multiplied by perpendicular height.* Use this to write down the area of each of the triangles shown below.





5. A(0, 0), B(4, −1), C(2, 3) and D(−2, 4) are the vertices of a quadrilateral.
Find the area of the quadrilateral by dividing it into the two triangles ABC and ACD.

Exercise 3.10

Area of Triangle with (0,0) = ½|$x_1y_2 - x_2y_1$|

6. Find the area of the quadrilateral with vertices A(0, 0), B(2, −3), C(4, 0) and D(0, 4).

7. The line $2x - y - 4 = 0$ intersects the x-axis at A and the y-axis at B.
Find the area of the △OAB, where O is the origin.

Exercise 3.10

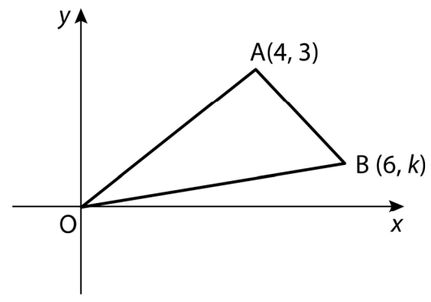
Area of Triangle with (0,0) = ½|$x_1y_2 - x_2y_1$|

8. Find the area of the triangle whose vertices are (0, 0), (1, 3) and (2, 6).
What conclusion can you draw from your answer?

Exercise 3.10

Area of Triangle with (0,0) = ½ $|x_1y_2 - x_2y_1|$

9. Find the value of k if the area of the given triangle is 7 square units.



Exercise 3.10 Answers

1. (i) $\frac{5}{2}$ sq. units (ii) $\frac{27}{2}$ sq. units
(iii) $\frac{5}{2}$ sq. units (iv) 5 sq. units
(v) 3 sq. units (vi) 9 sq. units

2. B′(−7, −2); C′(1, −2); 8 sq. units

3. (i) $\frac{9}{2}$ sq. units (ii) $\frac{33}{2}$ sq. units
(iii) 15 sq. units (iv) 4 sq. units

4. 14 sq. units; 14 sq. units

5. 14 sq. units

6. 14 sq. units

7. 4 sq. units

8. Not a triangle, i.e. a straight line

9. $k = 1$

Answers