Patterns and Sequences

Sequence

If we start at 0 and keep adding 4 we get the sequence

0, 4, 8, 12, 16, 20, 24, 28, 32, ...

A sequence is a set of numbers in a particular order.

Each number in the sequence is called a <u>term</u>

The sequence of the first five odd natural number

1, 3, 5, 7, 9

Term 1, Term 2, Term 3, T4, T5

The term to term rule

- Describes how to get from one term to the next.

Eg. The sequence is formed when you start at 30 and subtract 5 each time.

Ans = 30, 25, 20, 15, 10, 5, 0, −5

(each step: −5)

T_1 T_2 T_3 T_4 T_5 T_6 T_7 T_8

Eg2) A sequence starts at 7 and increases in steps of 4 each time

T_1 T_2 T_3 T_4 T_5 T_6

7, 11, 15, 19, 23, 27

(+4, +4, +4, 4, 4)

Q1) Write down the next three terms in the sequence and explain how the sequence is formed

2, 4, 6, 8, ... 10, 12, 14

+2 +2 +2 +2 2 2

Start at 2 and added 2 each time

Pg 277 Q2+3

Mon 5th/Nov Test corrections

1, 2, 5, 10, 17, 26

1 3 5 7 9

2, 6, 12, 20, 30, 42

4 6 8 10 12

2, 7, 14, 23

5 7 9

2 2

29, 47

first

76

$1^3, 2^3 3^3 4^3 5^3 6^3$

$$\begin{array}{r} 36 \\ {}^{3}6 \\ \hline 216 \end{array}$$

n^{th} term OR T_n OR term to term Rule

Eg) $T_n = n + 4$

first term $T_1 = (1) + 4 = 5$

2^{nd} term $T_2 = (2) + 4 = 6$

3^{rd} term $T_3 = (3) + 4 = 7$

100^{th} term $T_{100} = (100) + 4 = 104$

Pattern

5, 6, 7, 8

+1 +1 +1

first difference

Q1) Find the first 3 terms in the sequence, using the rule $4n+3$ => T_n n^{th} term rule

$4n+3$
nth

$T_1 = 4(1)+3$ First term $n=1$

$= 7$

$T_2 = 4(2)+3$ Second term $n=2$

11

$T_3 = 4(3)+3$ Third term $n=3$

15

Sequence 7, 11, 15

4 (nth), 4 (nth) common difference

$T_n = an+b$, a is the common difference

$T_n = \boxed{4}n+b$ nth term rule or T_n

$T_1 = 4(1) \pm \square = 7$

$4 + \boxed{3} = 7$

$T_n = 4n+3$

(Q2) Find the n^{th} term rule for the sequence

T_1

2, 5, 8, 11, ...

3 3 3 common difference

$T_n = an + b$

$T_n = 3n + b$

$T_{(1)} = 3(1) \pm \square = 2$

$3 \pm \square = 2$

$3 - 1 = 2$

n^{th} rule = $T_n = 3n - 1$

Q3) Find the n^{th} term rule and also T_{20} of the sequence

2, 7, 12, 17, ...

5 5 5 common diff

$T_n = 5n + b$

$T_1 = 5(1) \pm \square = 2$

$5 \pm \square = 2$

$5 - 3 = 2$ ✓

n^{th} term rule = $T_n = 5n - 3$

$T_{20} = 5(20) - 3$

$100 - 3 = 97$

$T_{20} = 97$.

C/W Pg 280 Q7 Find next two terms + Tn

i) 3, 5, 7, 9, 11, 13, 15

2 2 2

$T_n = an + b$

$T_1 = 2(1) \pm \square = 3$

$2 + 1 = 3$

$T_n = 2n + 1$

ii) 4, 7, 10, 13, 16, 19

3 3 3

$T_n = 3n \pm \square$

$T_1 = 3(1) \pm \square = 4$

$3 \pm \boxed{1} = 4$

$T_n = 3n + 1$

iii) 2, 6, 10, 14, 18, 22, 26

4 4 4 4

$T_n = 4n \pm \square$

$T_1 = 4(1) \pm \square = 2$

$4 - 2 = 2$

$T_n = 4n - 2$

iv) 5, 9, 13, 17, 21, 25

4 4 4 4

$T_n = 4n \pm \square$

$T_1 = 4(1) \pm \square = 5$

$4 \pm \boxed{1} = 5$

$T_n = 4n + 1$

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Q 2+3

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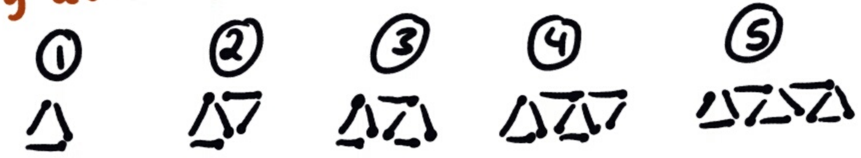
Q 4→7

Sequences from shapes

Pg 282 Q1



First six patterns : numbers generated by the matchsticks

T_1	T_2	T_3	T_4	T_5	T_6
3	5	7	9	11	13

+2 +2

iii) Find an expression for n for the number of matches in the n^{th} set of triangles

$T_n = an + b$ $a = 2$ common difference.

$T(1) = 2(1) \pm \square = 3$

$2 \pm \boxed{1} = 3$

$T_n = 2n + 1$

IV) How many matches are needed for the 50th set of triangles.

replaces n with 50 Ans

$T_{50} = 2(50) + 1 \Rightarrow 101$

Q2) T_1 6, 11, 16, 21, 26, 31

5 5 common difference.

$T_n = 5n + 1$ $\quad T_1 = 5(1) \pm \square = 6$

$5 + 1 = 6$

$T_n = 5n + 1$

20^{th} pattern $\Rightarrow n = 20$ $\quad T_{20} = 5(20) + 1$

$100 + 1$

$T_{20} = 101$ matches

$5n + 1 = 51$ find n

-1 | $5n = 50$ | -1

$\div 5$ | $n = 10$ | $\div 5$

Q3

Square	T_1 1	2	3	4	5	6
matches	4	7	10	13	16	19

3 3

n^{th} pattern

$T_n = 3n \pm \square$

$T_1 = 3(1) \pm \square = 4$

$3 + 1 = 4$

$T_n = 3n + 1$

$n = 50$ $T_{50} = 3(50) + 1$

$150 + 1$

$T_{50} = 151$ matches.

Q4) 1, 5, 9, 13

4 4

$T_n = 4n \pm \square$

$T_1 = 4(1) \pm \square = 1$

$4 - 3 = 1$

$T_n = 4n - 3$

$n = 30$

$T_{30} = 4(30) - 3$

$120 - 3$

$= 117$

$T_{30} = 117$ squares.

$4n - 3 = 77$ find

$+3$ | $4n = 80$ | $\pm 3n$

$\div 4$ | $n = 20$ | $\div 4$

$T_{20} = 77$ squares.

Q5)

Number	1	2	3	4	5	6	7
Matches	5	9	13	17	21	25	29

4 4

$\sim$

$T_n = 4n \pm \square$

$T_1 = 4(1) \pm \square = 5$

$4 + 1 = 5$

$T_n = 4n + 1$

$4n + 1 = 101$

-1 | $4n = 100$ | -1

$\div 4$ | $n = 25$ | $\div 4$

$T_{25} = 101$ matches

Q5 5, 7, 9, 11, 13 26

2 2 Common difference.

$T_n = 2a \pm D$

$T_1 = 2(1) \pm D = 5$

$2 + 3 = 5$

$T_n = 2n + 3$

$T_{10} = 2(10) + 3$

$20 + 3$

$T_{10} = 23$

$T_{100} = 2(100) + 3$

203

$2n + 3 = 101$

-3 | $2n = 98$ | -3

$\div 2$ | $n = 49$ | $\div 2$

Arithmatic Sequence - Linear

Log Tables Sequences + Series
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$$T_n = a + (n-1)d$$

a = first term
d = common difference.

Eg1) Find the n^{th} term of the arithmetic sequence.

1, 5, 9, 13, ...

4 4 4

first term = 1
$a = 1$
common difference = 4
$d = 4$

Sub values into formula

$T_n = 1 + (n-1)4$ (multiply) $\Rightarrow T_n = 4n - 3$

$1 + 4n - 4$

Eg2) Find T_n for the linear pattern

$-5, 0, 5, 10, \ldots$ $\quad a = -5$, $d = 5$

(+5, +5)

$T_n = a + (n-1)d$

$T_n = -5 + (n-1)5$ (multiply)

$-5 + 5n - 5$

$T_n = 5n - 10$

$T_n = 5n \pm \square$

$T_1 = 5(1) \pm \square = -5$

$5 - 10 = -5$

$T_n = 5n - 10$

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Q8) 4, 7, 10

3

$a = 4$

$d = 3$

$T_n = a + (n-1)d$

$T_n = 4 + (n-1)3$

$4 + 3n - 3$

$T_n = 3n + 1$

$T_{20} = 3(20) + 1$

$60 + 1$

$= 61$

Qa) $T_n = 4n - 1$

$T_1 = 4(1) - 1 = 3$

$T_2 = 4(2) - 1 = 7$ (+4)

$T_3 = 4(3) - 1 = 11$ (+4)

$a = 3$

$d = 4$

Q10) 2, 6, 10, ... $a = 2$, $d = 4$

(+4, +4)

$T_n = a + (n-1)d$

$2 + (n-1)4$

$2 + 4n - 4$

$T_n = 4n - 2$

$T_n = 46$

$4n - 2 = 46$

$4n = 48$ (+2) | $n = 12$ (÷4)

$T_{12} = 46$

11) 1, 3, 5, ...

+2 +2

$a = 1$

$d = 2$

$T_n = a + (n-1)d$

$T_n = 1 + (n-1)2$

$1 + 2n - 2$

$T_n = 2n - 1$

$T_n = 87$

$2n - 1 = 87$

$+1$ | $2n = 88$ | $+1$

$\div 2$ | $n = 44$ | $\div 2$

12) 1, 3, 6, 10, 15, 21

2 3 4 5 6 — first diff

1 1 1 1 — second constant = Quadratic

Finding the value of a and d in an arithmetic sequence.

Method

1. Sub given values into arithmetic sequence formula $T_n = a + (n-1)d$ to form two equations

2. Use simultaneous equations to find a and d.

Eg 1) In an arithmetic sequence
$T_4 = 14$ and $T_9 = 34$
Find the value of a and d.

① $T_4 = 14$
$T_n = a + (n-1)d$
$T_4 = a + (4-1)d = 14$
$a + (3)d = 14$
$a + 3d = 14$ ①
First equation

② $T_9 = 34$
$T_n = a + (n-1)d$
$T_9 = a + (9-1)d = 34$
$a + (8)d = 34$
$a + 8d = 34$ ②
Second Equation

Simultaneous Equation

multiply (-1)

① $a + 3d = 14$

② $a + 8d = 34$

$-a - 3d = -14$
$a + 8d = 34$
$5d = 20$
$d = 4$

Find a

① => $a + 3(4) = 14$
$a + 12 = 14$
$a = 2$

$T_n = a + (n-1)d$

$2 + (n-1)4$

$T_n = 2 + 4n - 4$

$T_n = 4n - 2$

$T_{13} = 4(13) - 2$

$52 - 2$

$T_{13} = 50$

HW
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Q3

H/W pg 289 Q3

$T_5 = 21$ and $T_{10} = 41$ Find a and d

T_n and T_{60}.